# DISCRETE MATHS (Q 6 & 7, PAPER 2)

## LESSON NO. 3: COUNTING

**2006**

7 (a) The password for a mobile phone consists of five digits.

(i) How many passwords are possible?

(ii) How many of these passwords start with a 2 and finish with an odd digit?

**SOLUTION**

**7 (a) (i)**

There are 10 ways to fill the first box and 10 ways to fill the second box and so on.

$10 \times 10 \times 10 \times 10 \times 10 = 100,000$

| 10 | × | 10 | × | 10 | × | 10 | × | 10 |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

**7 (a) (ii)**

There is one way to fill the first box, 10 ways the second, third and fourth boxes and 5 ways to fill the last box.

$1 \times 10 \times 10 \times 10 \times 5 = 5,000$

| 1 | × | 10 | × | 10 | × | 10 | × | 5 |
|---|---|---|---|---|---|---|---|---|
| **2** | | | | | | | | **Odd** |

**2005**

6 (a) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(i) the three digits are all different

(ii) the three digits are all the same?

**SOLUTION**

**6 (a) (i)**

There are 5 ways to fill the first box. Once this box is filled, there are 4 ways to fill the second box. Once the first two boxes are filled, there are 3 ways to fill the last box.

| 5 | × | 4 | × | 3 |
|---|---|---|---|---|
| | | | | |

No. of three-digit number all different $= 5 \times 4 \times 3 = 60$

**6 (a) (ii)**

There are 5 ways to fill the first box. Once this box is filled, there is only one way to fill the second box and the third box as the number in these boxes has to be the same as that in the first box.

| 5 | × | 1 | × | 1 |
|---|---|---|---|---|
| | | | | |

No. of three-digit number all the same $= 5 \times 1 \times 1 = 5$

**2005**

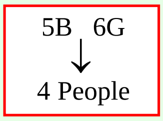
7 (a) (i) How many different groups of four can be selected from five boys and six girls?

(ii) How many of these groups consist of two boys and two girls?

**SOLUTION**

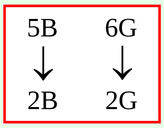
**7 (a) (i)**

No. of ways you can pick four from eleven people: ${}^{11}C_4 = 330$




**7 (a) (ii)**

No. of ways you can pick two boys from five boys and two girls from six girls: ${}^{5}C_2 \times {}^{6}C_2 = 10 \times 15 = 150$




**2004**

6 (a) A committee of five is to be selected from six students and three teachers.

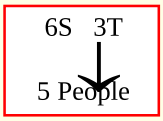
(i) How many different committees of five are possible?

(ii) How many of these possible committees have three students and two teachers?

**SOLUTION**

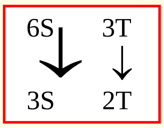
**6 (a) (i)**

No. of ways you can pick five from nine people: ${}^{9}C_5 = 126$




**6 (a) (ii)**

No. of ways you can pick three students from six students and two teachers from three teachers: ${}^{6}C_3 \times {}^{3}C_2 = 20 \times 3 = 60$



**2004**

7 (a) At the Olympic Games, eight lanes are marked on the running track. Each runner is allocated to a different lane. Find the number of ways in which the runners in a heat can be allocated to these lanes when there are

(i) eight runners in the heat

(ii) five runners in the heat and any five lanes may be used.

**SOLUTION**

**7 (a) (i)**

The first runner has a choice of 8 lanes, the second runner has a choice of 7 lanes once the first runner occupies a lane and so on.

$8\times7\times6\times5\times4\times3\times2\times1=40,320$

or $8!=40,320$

or ${}^{8}P_{8}=40,320$

8 × 7 × 6 × 5 × 4 3 2 1

**7 (a) (ii)**

The first runner has a choice of 8 lanes, the second runner has a choice of 7 lanes once the first runner occupies a lane and so on down to the fifth runner.

$8\times7\times6\times5\times4=6,720$

or ${}^{8}P_{5}=6,720$

8 × 7 × 6 × 5 × 4

**2003**

6 (a) Eight people, including Kieran and Anne, are available to form a committee. Five people must be chosen for the committee.

(i) In how many ways can the committee be formed if both Kieran and Anne must be chosen?

(ii) In how many ways can the committee be formed if neither Kieran nor Anne can be chosen?

**SOLUTION**

**6 (a) (i)**

If both Kieran and Anne must be chosen, then the number of ways of choosing three people from the remaining six people is ${}^{6}C_{3}=20$.

**6 (a) (ii)**

If neither Kieran nor Anne are to be chosen, then the number of ways of choosing five people from six people is ${}^{6}C_{5}=6$.

**2003**

7 (a) Five cars enter a car park. There are exactly five vacant spaces in the car park.

(i) In how many different ways can the five cars park in the vacant spaces?

(ii) Two of the cars leave the car park without parking. In how many different ways can the remaining three cars park in the five vacant spaces?

**SOLUTION**

**7 (a) (i)**

The first car has a choice of 5 spaces, the second car has a choice of 4 spaces once the first car occupies a space and so on down to the fifth car.

$5\times4\times3\times2\times1=120$

or ${}^5P_5=120$

or $5!=120$

5 × 4 × 3 × 2 × 1

**7 (a) (ii)**

The first car has a choice of 5 spaces, the second car has a choice of 4 spaces once the first car occupies a space and the third car has a choice of 3 spaces once the first two cars occupy their spaces.

$5\times4\times3=60$

or ${}^5P_3=60$

5 × 4 × 3

**2002**

6 (a) Nine friends wish to travel in a car. Only two of them, John and Mary, have licences to drive. Only five people can fit in the car (i.e. the driver and four others).
In how many ways can the group of five people be selected if

(i) both John and Mary are included

(ii) either John or Mary is included, but not both?

Later, another one of the nine friends, Anne, gets a driving licence.

(iii) The next time the journey is made, in how many ways can the group of five be chosen, given that at least one licenced driver must be included?

**SOLUTION**

**6 (a) (i)**

If John and Mary are included, you must calculate the number of ways you can pick three people from seven people.

${}^7C_3=35$

**6 (a) (ii)**

If John is included and Mary is not, you must calculate the number of ways you can pick four people from seven people *OR* if Mary is included and John is not, you must also calculate the number of ways you can pick four people from seven people.

$2\times{}^7C_4=70$

**Cont...**

**6 (a) (iii)**
No. of combinations with at least one qualified driver present
= Total no. of combinations – No. of combinations with no qualified driver
$= {}^9C_5 - {}^6C_5 = 126 - 6 = 120$

**2001**
6 (a) (i) How many different sets of three books or of four books can be selected from six different books?

(ii) How many of the above sets contain one particular book?

**SOLUTION**
**6 (a) (i)**
You need to choose sets of three books from six books *OR* sets of four books from six books.
${}^6C_3 + {}^6C_4 = 20 + 15 = 35$

**6 (a) (ii)**
If a particular book is included, this means you must choose sets of two books from five books *OR* sets of three books from five books.
${}^5C_2 + {}^5C_3 = 10 + 10 = 20$

**2001**
7 (a) (i) In how many different ways can four of the letters of the word FRIDAY be arranged if each letter is used no more than once in each arrangement?

(ii) How many of the above arrangements begin with the letter D and end with a vowel?

**SOLUTION**
**7 (a) (i)**
There are six ways to fill the first box. Once, this is filled there are five ways to fill the second box and so on.

$6 \times 5 \times 4 \times 3 = 360$

| | | | |
|---|---|---|---|
| | | | |

*OR*

The number of ways of arranging four letters from six letters is ${}^6P_4 = 360$

**7 (a) (ii)**

$1 \times 4 \times 3 \times 1 = 12$

| | | | |
|---|---|---|---|
| **D** | | | **A** |

$\oplus$

*OR*

$1 \times 4 \times 3 \times 1 = 12$

| | | | |
|---|---|---|---|
| **D** | | | **I** |

24

There are two possibilities for ending in a vowel. Consider each one separately and add together the two answers.
First possibility: There is only one way to fill the first box (with a **D**) and one way to fill the last box (with an **A**). This means there are four ways to fill the second box and three ways to fill the third box. There are 12 arrangements. Similarly there are 12 arrangement for the second possibility giving a total of 24 arrangements.