Discrete Maths (Q 6 & 7, Paper 2)

2010

6 (a) One bag contains four red discs and six blue discs.
Another bag contains five red discs and seven yellow discs.
One disc is drawn from each bag.
What is the probability that both discs are red?

(b) α and β are the roots of the quadratic equation $px^2 + qx + r = 0$.

$u_n = l\alpha^n + m\beta^n$, for all $n \in \mathbf{N}$.

Prove that $pu_{n+2} + qu_{n+1} + ru_n = 0$, for all $n \in \mathbf{N}$.

(c) In a café there are 11 seats in a row at the counter.
Six people are seated at random at the counter.

How much more likely is it that all six are seated together than that no two of them are seated together?

Solution

6 (a)

$$p(\text{Red and then Red}) = \frac{4}{10} \times \frac{5}{12} = \frac{1}{6}$$

Bag 1	**Bag 2**
4R, 6B	5R, 7Y

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$

6 (b)

Required to Prove: $u_n = l(\alpha)^n + m(\beta)^n$

Proof

$\Rightarrow pu_{n+2} = pl(\alpha)^{n+2} + pm(\beta)^{n+2} \quad = pl\alpha^2(\alpha)^n + pm\beta^2(\beta)^n$

$\Rightarrow qu_{n+1} = ql(\alpha)^{n+1} + qm(\beta)^{n+1} \quad = ql\alpha(\alpha)^n + qm\beta(\beta)^n$

$\Rightarrow ru_n = rl(\alpha)^n + rm(\beta)^n \quad = rl(\alpha)^n + rm(\beta)^n$

$\Rightarrow pu_{n+2} + qu_{n+1} + ru_n = (\alpha)^n l(p\alpha^2 + q\alpha + r) + (\beta)^n m(p\beta^2 + q\beta + r)$

$= (\alpha)^n l(0) + (\beta)^n m(0) = 0 + 0 = 0$

once α, β are the roots of $px^2 + qx + r = 0$.

6 (c)

6 people: A, B, C, D, E, F

11 seats

A		B		C		D		E		F

If the six people are not to sit together then they need to occupy the seats as shown above. There is no other seating possibility if all 6 people are to remain apart.

A	B	C	D	E	F					
	A	B	C	D	E	F				
		A	B	C	D	E	F			
			A	B	C	D	E	F		
				A	B	C	D	E	F	
					A	B	C	D	E	F

If they are to sit together, there are six different possibilities for occupying the seats as shown above.

Therefore, it is 6 times more likely that all six are seated together rather than apart.

7 (a) A password for a website consists of capital letters A, B, C,. … Z and/or digits 0, 1, 2, …. 9.
The password has four such characters and starts with a letter. For example, BA7A, C999 and DGKK are allowed, but 7DCA is not.

Show that there are more than a million possible passwords.

(b) Karen is about to sit an examination at the end of an English course. The course has twenty prescribed texts. Six of these are novels, four are plays and ten are poems.

The examination consists of a question on one of the novels, a question on one of the plays and a question on one of the poems.

Karen has studied four of the novels, three of the plays and seven of the poems.
Find the probability that:

(i) Karen has studied all three of the texts on the examination

(ii) Karen has studied none of the texts on the examination

(iii) Karen has studied at least two of the texts on the examination.

(c) The real numbers a, $2a$, $3a$, $4a$ and $5a$ have mean μ and standard deviation σ.

(i) Express μ and σ in terms of a.

(ii) Hence write down, in terms of a, the mean and the standard deviation of

$$3a + 5,\ 6a + 5,\ 9a + 5,\ 12a + 5,\ 15a + 5.$$

SOLUTION

7 (a)

$$26 \times 36 \times 36 \times 36 = 1{,}213{,}056$$

7 (b)

20 texts: 6 novels, 4 plays, 10 poems
Exam: 3 questions, one on a novel, one on a play and one on a poem.
Karen has studied: 4 novels, 3 plays, 7 poems

7 (b) (i)

p(Karen has studied all questions on the exam) $= \frac{4}{6} \times \frac{3}{4} \times \frac{7}{10} = \frac{7}{20}$

7 (b) (ii)

p(Karen has studied none of the questions on the exam) $= \frac{2}{6} \times \frac{1}{4} \times \frac{3}{10} = \frac{1}{40}$

7 (b) (iii)

p(Karen has studied at least two) = p(Karen has studied two) + p(Karen has studied all 3)
= p(Karen has studied a novel and play but not a poem)
+ p(Karen has studied a novel and poem but not a play)
+ p(Karen has studied a play and poem but not a novel)
+ p(Karen has studied all 3)

$$= \tfrac{4}{6}\times\tfrac{3}{4}\times\tfrac{3}{10}+\tfrac{4}{6}\times\tfrac{1}{4}\times\tfrac{7}{10}+\tfrac{2}{6}\times\tfrac{3}{4}\times\tfrac{7}{10}+\tfrac{7}{20}=\tfrac{19}{24}$$

7 (c) (i)

Numbers: a, $2a$, $3a$, $4a$, $5a$

$$\mu = \frac{a+2a+3a+4a+5a}{5} = \frac{15a}{5} = 3a$$

$$\bar{x} = \frac{x_1 + x_2 + \dots\dots\dots\dots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$

x	d	d^2
a	$-2a$	$4a^2$
$2a$	$-a$	a^2
$3a$	0	0
$4a$	a	a^2
$5a$	$2a$	$4a^2$
		$10a^2$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$

$$\sigma = \sqrt{\frac{10a^2}{5}} = \sqrt{2a^2} = \sqrt{2}a$$

7 (c) (ii)

Numbers: $3a + 5$, $6a + 5$, $9a + 5$, $12a + 5$, $15a + 5$

Mean $= 3\mu + 5 = 3(3a) + 5 = 9a + 5$

Standard deviation $= 3\sigma = 3\sqrt{2}a$