Discrete Maths (Q 6 & 7, Paper 2)

1997

6 (a) How many different four digit numbers greater that 5000 can be formed from the digits 2, 4, 5, 8, 9 if each digit can be used only once in any given number?
How many of these numbers are odd?

(b) Solve the difference equation

$$u_{n+2} - 4u_{n+1} + u_n = 0,\ n \geq 0$$

where $u_0 = 4$ and $u_1 = 8$.

(c) The following data give the age and gender of twenty five pupils in a class on a given day:

	Boys	Girls
Number of pupils aged sixteen years	5	7
Number of pupils aged seventeen years	7	6

(i) One of the pupils is picked at random. What is the probability that a boy aged sixteen years or a girl aged seventeen years is picked?

(ii) Each pupil in the class is given his/her examination results. Only three pupils scored full marks. Determine the probability that these three pupils are of the same age and the same gender.

Solution

6 (a)

There are 3 ways to fill the first box (with a 5, 8 or 9). Once this is filled there are 4 numbers left to fill the second box. Once the first 2 boxes are filled there are 3 numbers to fill the third box and so on.

Number of arrangements = 3 × 4 × 3 × 2 = 72

An odd number ends in either a 5 or a 9. Therefore, there are 2 ways to fill the last box. Once this is filled, there are just 2 ways to fill the first box (with an 8 and the odd number that is not in the last box).
Once these boxes are filled there are 3 numbers left to fill the second box and so on.

Number of arrangements = 2 × 4 × 3 × 2 = 48

Odd

6 (b)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $u_{n+2} - 4u_{n+1} + u_n = 0$

2. $x^2 - 4x + 1 = 0$

3. $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} = \dfrac{4 \pm \sqrt{12}}{2} = \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$\therefore \alpha = 2 + \sqrt{3},\ \beta = 2 - \sqrt{3}$

4. $u_n = l(2+\sqrt{3})^n + m(2-\sqrt{3})^n$

5. $u_0 = 4 \Rightarrow l + m = 4 \ldots(\mathbf{1})$

$u_1 = 8 \Rightarrow l(2+\sqrt{3}) + m(2-\sqrt{3}) = 8 \ldots.(\mathbf{2})$

Solve Eqns. (**1**) and (**2**) simultaneously:

$$l + m = 4 \ldots(\mathbf{1})(\times -(2-\sqrt{3})$$
$$l(2+\sqrt{3}) + m(2-\sqrt{3}) = 8 \ldots.(\mathbf{2})$$

$$-(2-\sqrt{3})l - (2-\sqrt{3})m = -4(2-\sqrt{3})$$
$$(2+\sqrt{3})l + (2-\sqrt{3})m = 8$$
$$(2+\sqrt{3}-2+\sqrt{3})l \quad = 8 - 8 + 4\sqrt{3} \Rightarrow 2\sqrt{3}l = 4\sqrt{3} \Rightarrow l = 2$$

Substitute this value of l into Eqn. (**1**): $m = 2$

Answer: $u_n = 2(2+\sqrt{3})^n + 2(2-\sqrt{3})^n$

6 (c) (i)

	Boys	Girls
Number of pupils aged sixteen years	**5**	7
Number of pupils aged seventeen years	7	**6**

p(Boy aged 16 or Girl aged 17) $= \frac{11}{25}$

6 (c) (ii)

Forget about the exam marks. Pose the question like this:
What is the probability of picking randomly 3 pupils of the same age and gender?
p(3 boys aged 16 OR 3 boys aged 17 OR 3 girls aged 16 OR 3 girls aged 17)

$$= \frac{5}{25}\times\frac{4}{24}\times\frac{3}{23}+\frac{7}{25}\times\frac{6}{24}\times\frac{5}{23}+\frac{7}{25}\times\frac{6}{24}\times\frac{5}{23}+\frac{6}{25}\times\frac{5}{24}\times\frac{4}{23}=\frac{1}{23}$$

7 (a) In how many ways can a group of four people be selected from three men and four women?
In how many of these groups are there more women than men?

(b) Two persons look at the letters in the word DISCOVERY.
Independently of one another, each person writes down two of the letters from the word DISCOVERY.
What is the probability that
(i) one person writes down two vowels and the other person two consonants?
(ii) the two persons write down different letters, that is, they have no letters in common?

(c) The data in the set (1, 2, 5, x, y} have a mean of 5.
Express, in terms of x,
(i) y
(ii) σ, the standard deviation of the data.

If the standard deviation is $\sqrt{\frac{99}{10}}$, find the value of x and the value of y.

SOLUTION

7 (a)

The number of selections of n different objects taking r at a time $= {}^{n}C_r$ 11

3M 4W
↓
4 Places

$${}^{7}C_4 = \frac{7\times6\times5\times4}{4\times3\times2\times1} = 35$$

More women than men can mean 4 women and no men OR 3 women and 1 man.

4 Women and 0 Men $= {}^{4}C_4 \times {}^{3}C_0 = 1$

3M 4W
↓ ↓
0M 4W

\+

3 Women and 1 Woman $= {}^{4}C_3 \times {}^{3}C_1 = 12$

13

3M 4W
↓ ↓
1M 3W

Note: OR means add.

7 (b) (i) DISCOVERY

Vowels: IOE (3 letters)
Consonant: DSCVRY (6 letters)

Person 1		Person 2
Vowel and vowel	AND	Consonant and consonant

OR

Person 1		Person 2
Consonant and consonant	AND	Vowel and vowel

The probability of the first person picking a vowel is $\frac{3}{9}$ as there are 3 vowels to pick from 9 letters. The probability of the first person picking a second vowel is $\frac{2}{8}$ as there are 2 vowels left to pick from the remaining 8 letters.
The second person has no idea what the first person has picked. The probability of the second person picking a consonant is $\frac{6}{9}$ as there are 6 consonants to pick from 9 letters. The probability of the second person picking a second consonant is $\frac{5}{8}$ as there are 5 consonants left to pick from the remaining 8 letters.
OR the first person could have picked 2 consonants and the second person 2 vowels. Carry out the same procedure. It turns out to be the same. Add both probabilities together as OR means add.

p(Vowel, Vowel, Consonant, Consonant) OR p(Consonant, Consonant, Vowel, Vowel)
$= \frac{3}{9} \times \frac{2}{8} \times \frac{6}{9} \times \frac{5}{8} + \frac{6}{9} \times \frac{5}{8} \times \frac{3}{9} \times \frac{2}{8} = \frac{5}{72}$

7 (b) (ii)

The first person writes down 2 letters. She can see what she is writing down so there is a certainty that she will write down 2 different letters. The probability that the first person writes down 2 different letters is 1.
The second person cannot see what she has written down. The probability that the second person writes down a letter different to those letters written by the first person is $\frac{7}{9}$ as there are 7 different letters to pick from 9 letters.
The probability of the second person picking a second letter that is different to the other letters is $\frac{6}{8}$ as the second person can pick 6 letters from the remaining 8 letters.

p(2 persons write different letters) $= 1 \times \frac{7}{9} \times \frac{6}{8} = \frac{7}{12}$

7 (c) (i)

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots\ldots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$ 3

$$\frac{1+2+5+x+y}{5} = 5 \Rightarrow 1+2+5+x+y = 25$$

$$\Rightarrow y = 17 - x$$

7 (c) (ii)

x	d	d^2
1	-4	16
2	-3	9
5	0	0
x	$(x-5)$	$(x-5)^2$
y	$(y-5)$	$(y-5)^2$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ 6

where deviation $d = (x - \bar{x}) = (\text{Number} - \text{Mean})$

$$\sigma = \sqrt{\frac{16+9+0+(x-5)^2+(y-5)^2}{5}} = \sqrt{\frac{99}{10}}$$

$$\Rightarrow \frac{16+9+0+(x-5)^2+(12-x)^2}{5} = \frac{99}{10}$$

$$\Rightarrow \frac{25+x^2-10x+25+144-24x+x^2}{5} = \frac{99}{10}$$

$$\Rightarrow 2x^2 - 34x + 194 = \frac{99}{2}$$

$$\Rightarrow 4x^2 - 68x + 388 = 99$$

$$\Rightarrow 4x^2 - 68x + 289 = 0$$

$$\Rightarrow (2x-17)(2x-17) = 0$$

$$\therefore x = \tfrac{17}{2}$$

$$\Rightarrow y = 17 - x = 17 - \tfrac{17}{2} = \tfrac{17}{2}$$