DISCRETE MATHS (Q 6 & 7, PAPER 2)

2006

6 (a) (i) How many different teams of three people can be chosen can be chosen from a panel of six boys and five girls?

(ii) If the team is chosen at random, find the probability that it consists of girls only?

6 (b) (i) Solve the difference equation $6u_{n+2} - 7u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 8$ and $u_1 = 3$.

(ii) Verify that the solution to part (**i**) also satisfies the difference equation $6u_{n+1} - u_n - 10 = 0$.

6 (c) There are thirty days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.

(i) What is the probabilty that all seven students have the same birthday?

(ii) What is the probability that all seven students have different birthdays?

(iii) Show that the probability that at least two have the same birthday is greater than 0·5?

SOLUTION

6 (a) (i)

You are asked to choose teams of 3 people from 11 people.

6B 5G
↓
3 Places

$$^{11}C_3 = \frac{11 \times 10 \times 9}{3 \times 2 \times 1} = 165$$

6 (a) (ii)

The number of ways teams of 3 girls can be chosen from 5 girls:

6B	5G
↓	↓
0 Places	3 Places

$$^{5}C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

$p(\text{Choosing a girls team}) = \frac{10}{165} = \frac{2}{33}$

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)} \quad \ldots\ldots \quad 12$$

6 (b) (i)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find *l*, *m* using extra conditions (boundary conditions).

1. $6u_{n+2} - 7u_{n+1} + u_n = 0$

2. $6x^2 - 7x + 1 = 0$

3. $(x-1)(6x-1) = 0 \Rightarrow \alpha = 1,\ \beta = \frac{1}{6}$

4. $u_n = l(1)^n + m(\frac{1}{6})^n = l + m(\frac{1}{6})^n$

5. $u_0 = 8 = l + m$ and $u_1 = 3 = l + m(\frac{1}{6}) \Rightarrow 18 = 6l + m$

Solving these equations simultaneously $\Rightarrow l = 2,\ m = 6$

Ans: $u_n = 2 + 6(\frac{1}{6})^n = 2 + (\frac{1}{6})^{n-1}$

6 (b) (ii)

Substitute the solution into $6u_{n+1} - u_n - 10 = 0$.

$u_n = 2 + (\frac{1}{6})^{n-1}$

$u_{n+1} = 2 + (\frac{1}{6})^n$

$\Rightarrow 6u_{n+1} - u_n - 10 = 6(2 + (\frac{1}{6})^n) - (2 + (\frac{1}{6})^{n-1}) - 10$

$= 12 + (\frac{1}{6})^{n-1} - 2 - (\frac{1}{6})^{n-1} - 10 = 0$

6 (c) (i)

p(A born on a day in June AND THEN B born on the same day as A AND THEN C is born on the same day as A AND THEN D is born on the same day as A AND THEN E is born on the same day as A AND THEN F is born on the same day as A AND THEN G is born on the same day as A)

$$= \frac{30}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} = \frac{1}{30^6}$$

6 (c) (ii)

p(A born on a weekday AND THEN B born on a different day to A, AND THEN C born on a different day to B and A, AND THEN D born on a different day to C and B and A, AND THEN E born on a different day to D and C and B and A, AND THEN F born on a different day to E and D and C and B and A, AND THEN G born on a different day to F and E and D and C and B and A)

$$= \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30} \times \frac{25}{30} \times \frac{24}{30} = \frac{2639}{5625}$$

6 (c) (iii)

At least one = 1 − p(None)
At least two = 1 − p(None or one) etc... ...... **15**

p(At least two have the same birthday = 1 − p(All seven have different birthdays)

$$=1-\frac{2639}{5625}=\frac{2986}{5625}=0\cdot53>0\cdot5$$

7 (a) The password for a mobile phone consists of five digits.

(i) How many passwords are possible?

(ii) How many of these passwords start with a 2 and finish with an odd digit?

7 (b) For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as the winning combination.

(i) How many different combinations are possible?

(ii) How many of all the possible combinations will match exactly three numbers with the winning combination?

(iii) How many of all the possible combinations will match exactly three numbers with the winning combination?

(iv) Show that the probability of matching at least three numbers with the winning combination is approximately 0·014.

7 (c) The mean of the integers form $-n$ to n, inclusive, is 0. Show that the standard deviation is $\sqrt{\frac{n(n+1)}{3}}$.

Solution

7 (a) (i)

There are 10 ways to fill the first box and 10 ways to fill the second box and so on.

$10\times10\times10\times10\times10=100,000$

10	× 10	× 10	× 10	× 10

7 (a) (ii)

There is one way to fill the first box, 10 ways the second, third and fourth boxes and 5 ways to fill the last box.

$1\times10\times10\times10\times5=5,000$

1	× 10	× 10	× 10	× 5
2				**Odd**

7 (b) (i)

How many ways can you pick 5 number from 35 numbers: ${}^{35}C_5 = 324,632$

7 (b) (ii)

There are 5 winning (**W**) numbers and 30 non-winning (**NW**) numbers. To match 4 winning numbers, you need to pick 4 from 5 winning numbers and 1 from 30 non-winning numbers.

5**W**	30**NW**
↓	↓
4 Nos.	1 No

No. of match 4 combinations: ${}^{5}C_4 \times {}^{30}C_1 = 5 \times 30 = 150$

7 (b) (iii)

5**W**	30**NW**
↓	↓
3 Nos.	2 Nos.

No. of match 3 combinations: ${}^{5}C_3 \times {}^{30}C_2 = 10 \times 435 = 4,350$

7 (b) (iii)

p(Matching at least 3 Nos.) = p(Matching 3) + p(Matching 4) + p(Matching 5)

$$= \frac{4350}{324632} + \frac{150}{324632} + \frac{1}{324632} = \frac{4501}{324632} = 0\cdot 014$$

7 (c)

Write out the integers as a list:

$-n,\ -n+1,\ -n+2, \ldots\ldots, -2,\ -1,\ 0,\ 1,\ 2, \ldots\ldots,\ n-2,\ n-1,\ n$

As you can see, the mean of these numbers is zero.

The number of numbers, $N = 2n + 1$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}} \quad \ldots\ldots\ \textbf{6}$$

where deviation $d = (x - \bar{x}) = (\text{Number} - \text{Mean})$

x	d	d^2
$-n$	$-n$	n^2
$-n+1$	$-n+1$	$(n-1)^2$
$-n+2$	$-n+2$	$(n-2)^2$
⋮	⋮	⋮
-2	-2	4
-1	-1	1
0	0	0
1	1	1
2	2	4
⋮	⋮	⋮
$n-2$	$n-2$	$(n-2)^2$
$n-1$	$n-1$	$(n-1)^2$
n	n	n^2
		$\sum d^2 = 2\sum_{r=1}^{n} r^2$

$$\sigma = \sqrt{\frac{2\sum_{r=1}^{n} r^2}{(2n+1)}}$$

$$\sum_{r=1}^{n} r^2 = S_n = 1^2 + 2^2 + \ldots\ldots + n^2 = \tfrac{n}{6}(n+1)(2n+1) \quad \ldots\ \textbf{8}$$

$$\Rightarrow \sigma = \sqrt{\frac{2 \times \frac{n}{6}(2n+1)(n+1)}{(2n+1)}} = \sqrt{\frac{n(n+1)}{3}}$$