

DISCRETE MATHS (Q 6 & 7, PAPER 2)

2006

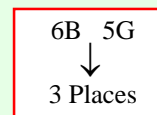
- 6 (a) (i) How many different teams of three people can be chosen from a panel of six boys and five girls?
- (ii) If the team is chosen at random, find the probability that it consists of girls only?
- 6 (b) (i) Solve the difference equation $6u_{n+2} - 7u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 8$ and $u_1 = 3$.
- (ii) Verify that the solution to part (i) also satisfies the difference equation $6u_{n+1} - u_n - 10 = 0$.
- 6 (c) There are thirty days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.
- (i) What is the probability that all seven students have the same birthday?
- (ii) What is the probability that all seven students have different birthdays?
- (iii) Show that the probability that at least two have the same birthday is greater than 0.5?

SOLUTION

6 (a) (i)

You are asked to choose teams of 3 people from 11 people.

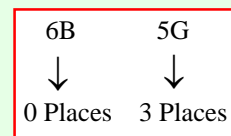
$${}^{11}C_3 = \frac{11 \times 10 \times 9}{3 \times 2 \times 1} = 165$$



6 (a) (ii)

The number of ways teams of 3 girls can be chosen from 5 girls:

$${}^5C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$



$$p(\text{Choosing a girls team}) = \frac{10}{165} = \frac{2}{33}$$

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)} \dots\dots$$

6 (b) (i)

$$u_n = l(\alpha)^n + m(\beta)^n \dots\dots \mathbf{1}$$

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β .
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $6u_{n+2} - 7u_{n+1} + u_n = 0$

2. $6x^2 - 7x + 1 = 0$

3. $(x-1)(6x-1) = 0 \Rightarrow \alpha = 1, \beta = \frac{1}{6}$

4. $u_n = l(1)^n + m(\frac{1}{6})^n = l + m(\frac{1}{6})^n$

5. $u_0 = 8 = l + m$ and $u_1 = 3 = l + m(\frac{1}{6}) \Rightarrow 18 = 6l + m$

Solving these equations simultaneously $\Rightarrow l = 2, m = 6$

Ans: $u_n = 2 + 6(\frac{1}{6})^n = 2 + (\frac{1}{6})^{n-1}$

6 (b) (ii)

Substitute the solution into $6u_{n+1} - u_n - 10 = 0$.

$$u_n = 2 + (\frac{1}{6})^{n-1}$$

$$u_{n+1} = 2 + (\frac{1}{6})^n$$

$$\Rightarrow 6u_{n+1} - u_n - 10 = 6(2 + (\frac{1}{6})^n) - (2 + (\frac{1}{6})^{n-1}) - 10$$

$$= 12 + (\frac{1}{6})^{n-1} - 2 - (\frac{1}{6})^{n-1} - 10 = 0$$

6 (c) (i)

p (A born on a day in June AND THEN B born on the same day as A AND THEN C is born on the same day as A AND THEN D is born on the same day as A AND THEN E is born on the same day as A AND THEN F is born on the same day as A AND THEN G is born on the same day as A)

$$= \frac{30}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} \times \frac{1}{30} = \frac{1}{30^6}$$

6 (c) (ii)

p (A born on a weekday AND THEN B born on a different day to A, AND THEN C born on a different day to B and A, AND THEN D born on a different day to C and B and A, AND THEN E born on a different day to D and C and B and A, AND THEN F born on a different day to E and D and C and B and A, AND THEN G born on a different day to F and E and D and C and B and A)

$$= \frac{30}{30} \times \frac{29}{30} \times \frac{28}{30} \times \frac{27}{30} \times \frac{26}{30} \times \frac{25}{30} \times \frac{24}{30} = \frac{2639}{5625}$$

6 (c) (iii)

At least one = $1 - p(\text{None})$
At least two = $1 - p(\text{None or one})$ etc... .. **15**

$$p(\text{At least two have the same birthday}) = 1 - p(\text{All seven have different birthdays})$$
$$= 1 - \frac{2639}{5625} = \frac{2986}{5625} = 0.53 > 0.5$$

7 (a) The password for a mobile phone consists of five digits.

- (i) How many passwords are possible?
- (ii) How many of these passwords start with a 2 and finish with an odd digit?

7 (b) For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as the winning combination.

- (i) How many different combinations are possible?
- (ii) How many of all the possible combinations will match exactly three numbers with the winning combination?
- (iii) How many of all the possible combinations will match exactly three numbers with the winning combination?
- (iv) Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.

7 (c) The mean of the integers form $-n$ to n , inclusive, is 0. Show that the standard

deviation is $\sqrt{\frac{n(n+1)}{3}}$.

SOLUTION

7 (a) (i)

There are 10 ways to fill the first box and 10 ways to fill the second box and so on.

$$10 \times 10 \times 10 \times 10 \times 10 = 100,000$$

$$10 \times 10 \times 10 \times 10 \times 10$$

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7 (a) (ii)

There is one way to fill the first box, 10 ways the second, third and fourth boxes and 5 ways to fill the last box.

$$1 \times 10 \times 10 \times 10 \times 5 = 5,000$$

$$1 \times 10 \times 10 \times 10 \times 5$$

2				Odd
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7 (b) (i)

How many ways can you pick 5 number from 35 numbers: ${}^{35}C_5 = 324,632$

7 (b) (ii)

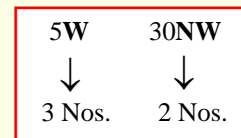
There are 5 winning (W) numbers and 30 non-winning (NW) numbers. To match 4 winning numbers, you need to pick 4 from 5 winning numbers and 1 from 30 non-winning numbers.



No. of match 4 combinations: ${}^5C_4 \times {}^{30}C_1 = 5 \times 30 = 150$

7 (b) (iii)

No. of match 3 combinations: ${}^5C_3 \times {}^{30}C_2 = 10 \times 435 = 4,350$



7 (b) (iii)

$p(\text{Matching at least 3 Nos.}) = p(\text{Matching 3}) + p(\text{Matching 4}) + p(\text{Matching 5})$

$$= \frac{4350}{324632} + \frac{150}{324632} + \frac{1}{324632} = \frac{4501}{324632} = 0.014$$

7 (c)

Write out the integers as a list:

$-n, -n+1, -n+2, \dots, -2, -1, 0, 1, 2, \dots, n-2, n-1, n$

As you can see, the mean of these numbers is zero.

The number of numbers, $N = 2n + 1$

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}} \dots\dots \mathbf{6}$$

x	d	d^2
$-n$	$-n$	n^2
$-n+1$	$-n+1$	$(n-1)^2$
$-n+2$	$-n+2$	$(n-2)^2$
■	■	■
■	■	■
-2	-2	4
-1	-1	1
0	0	0
1	1	1
2	2	4
■	■	■
■	■	■
$n-2$	$n-2$	$(n-2)^2$
$n-1$	$n-1$	$(n-1)^2$
n	n	n^2
		$\sum d^2 = 2 \sum_{r=1}^n r^2$

where deviation $d = (x - \bar{x}) = (\text{Number} - \text{Mean})$

$$\sigma = \sqrt{\frac{2 \sum_{r=1}^n r^2}{(2n+1)}}$$

$$\sum_{r=1}^n r^2 = S_n = 1^2 + 2^2 + \dots + n^2 = \frac{n}{6}(n+1)(2n+1) \dots \mathbf{8}$$

$$\Rightarrow \sigma = \sqrt{\frac{2 \times \frac{n}{6}(2n+1)(n+1)}{(2n+1)}} = \sqrt{\frac{n(n+1)}{3}}$$