DISCRETE MATHS (Q 6 & 7, PAPER 2)

2005

6 (a) How many three-digit numbers can be formed from the digits 1, 2, 3, 4, 5, if

(i) the three digits are all different

(ii) the three digits are all the same?

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} - 8u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2$.

(ii) Verify that your solution gives the correct value for u_2.

6 (c) Nine cards are numbered from 1 to 9. Three cards are drawn at random from the nine cards.

(i) Find the probability that the card numbered 8 is not drawn.

(ii) Find the probability that all three cards drawn have odd numbers.

(iii) Find the probability that the sum of the numbers on the cards drawn is greater than the sum of the numbers on the cards not drawn.

SOLUTION

6 (a) (i)

There are 5 ways to fill the first box. Once this box is filled, there are 4 ways to fill the second box. Once the first two boxes are filled, there are 3 ways to fill the last box.

5 × 4 × 3

No. of three-digit number all different $= 5 \times 4 \times 3 = 60$

6 (a) (ii)

There are 5 ways to fill the first box. Once this box is filled, there is only one way to fill the second box and the third box as the number in these boxes has to be the same as that in the first box.

5 × 1 × 1

No. of three-digit number all the same $= 5 \times 1 \times 1 = 5$

6 (b) (i)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $u_{n+2} - 4u_{n+1} - 8u_n = 0$

2. $x^2 - 4x - 8 = 0$

3. $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2} = \dfrac{4 \pm \sqrt{48}}{2} = \dfrac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

$\Rightarrow \alpha = 2 + 2\sqrt{3},\ \beta = 2 - 2\sqrt{3}$

4. $u_n = l(2+2\sqrt{3})^n + m(2-2\sqrt{3})^n$

5. $u_0 = l + m = 0$ ....(**1**) and $u_1 = l(2+2\sqrt{3})^1 + m(2-2\sqrt{3})^1 = 2$ ....(**2**)

From equation (**1**) $\Rightarrow m = -l$

Substitute into equation (**2**): $\Rightarrow l(2+2\sqrt{3}) - l(2-2\sqrt{3}) = 2 \Rightarrow 2l + 2l\sqrt{3} - 2l + 2l\sqrt{3} = 2$

$\Rightarrow 4l\sqrt{3} = 2 \Rightarrow l = \dfrac{1}{2\sqrt{3}} = \dfrac{1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{6}$

$\therefore l = \dfrac{\sqrt{3}}{6},\ m = -\dfrac{\sqrt{3}}{6}$

Ans: $u_n = \frac{\sqrt{3}}{6}(2+2\sqrt{3})^n - \frac{\sqrt{3}}{6}(2-2\sqrt{3})^n$

6 (b) (ii)

Put n = 0 into the difference equation: $u_2 - 4u_1 - 8u_0 = 0 \Rightarrow u_2 = 4(2) + 8(0) = 8$

Put n = 2 into the solution:

$u_2 = \frac{\sqrt{3}}{6}(2+2\sqrt{3})^2 - \frac{\sqrt{3}}{6}(2-2\sqrt{3})^2 \Rightarrow u_2 = \frac{\sqrt{3}}{6}(4 + 8\sqrt{3} + 12 - 4 + 8\sqrt{3} - 12)$

$\Rightarrow u_2 = \frac{\sqrt{3}}{6}(4 + 8\sqrt{3} + 12 - 4 + 8\sqrt{3} - 12) = \frac{\sqrt{3}}{6}(16\sqrt{3}) = 8$

You have verified that you get the correct solution for u_2.

6 (c)

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

6 (c) (i)

Find the probability of a particular combination and then calculate the number of ways in which that combination can take place.

$p(\text{Not 8, Not 8, Not 8}) = \frac{8}{9}\times\frac{7}{8}\times\frac{6}{7}=\frac{6}{9}=\frac{2}{3}$

N8	**N8**	**N8**

6 (c) (ii)

$p(\text{Odd, Odd, Odd}) = \frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}=\frac{5}{42}$

Odd	**Odd**	**Odd**

6 (c) (iii)

Write out all the possibilities:

7	**8**	**9**
6	**8**	**9**
5	**8**	**9**
4	**8**	**9**

What is the probability of picking the first combination?

$p(7, 8, 9) = \frac{1}{9}\times\frac{1}{8}\times\frac{1}{7}\times 3=\frac{1}{168}$

There are four such possibilities.

p(Sum of cards drawn is greater than those not drawn)

$=\frac{1}{168}\times 4=\frac{1}{42}$

7 (a) (i) How many different groups of four can be selected from five boys and six girls?

(ii) How many of these groups consist of two boys and two girls?

7 (b) There are sixteen discs in a board-game: five blue, three green, six red and two yellow. Four discs are chosen at random. What is the probability that

(i) the four discs are blue

(ii) the four discs are the same colour

(iii) all four discs are different in colour

(iv) two of the discs are blue and two are not blue?

7 (c) On 1st September 2003 the mean age of the first-year students in a school is 12·4 years and the standard deviation is 0·6 years. One year later all of these students have moved into second year and no other students have joined them.

(i) State the mean and the standard deviation of the ages of these students on 1st September 2004. Give a reason for each answer.

A new group of first-year students begins on 1st September 2004. This group has a similar age distribution and is of a similar size to the first-year group of September 2003.

(ii) State the mean age of the combined group of the first-year and second-year students on 1st September 2004.

(iii) State whether the standard deviation of the ages of this combined group is less than, equal to, or greater than 0·6 years. Give a reason for your answer.

Solution

7 (a) (i)

No. of ways you can pick four from eleven people: ${}^{11}C_4 = 330$

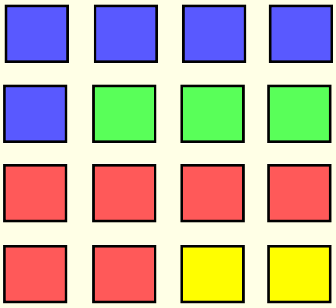
5B 6G
↓
4 People

7 (a) (ii)

No. of ways you can pick two boys from five boys and two girls from six girls: ${}^{5}C_2 \times {}^{6}C_2 = 10 \times 15 = 150$

5B → 2B
6G → 2G

7 (b)



7 (b) (i)

$p(\text{B, B, B, B}) = \frac{5}{16}\times\frac{4}{15}\times\frac{3}{14}\times\frac{2}{13} = \frac{1}{364}$

7 (b) (ii)

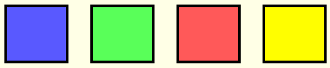
p(All the same colour) = p(B, B, B, B) + p(R, R, R, R)

$= \frac{1}{364} + \frac{6}{16}\times\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13} = \frac{1}{364} + \frac{3}{364} = \frac{4}{364} = \frac{1}{91}$

7 (b) (iii)

To find the probability that the four discs drawn are different in colour, find out the probability of picking four definite colours in a definite order and then multiply this probability by the number of combinations of the four colours.

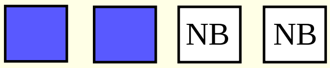
$p(\text{B, G, R, Y}) = \frac{5}{16}\times\frac{3}{15}\times\frac{6}{14}\times\frac{2}{13}\times 24 = \frac{9}{91}$



No. of combinations = 4! = 24

7 (b) (iii)

p(B, B, Not Blue (NB), NB) $= \frac{5}{16}\times\frac{4}{15}\times\frac{11}{14}\times\frac{10}{13}\times 6 = \frac{55}{182}$



No. of combinations $= \frac{4!}{2!2!} = 6$

7 (c) (i)

Mean = 13·4 years

As all the students are one year older, the mean is increased by one.

Standard deviation = 0·6 years

The spread of ages about the mean is still the same.

7 (c) (ii)

$$\text{Mean} \approx \frac{12\cdot 4 + 13\cdot 4}{2} = 12\cdot 9$$

7 (c) (iii)

Standard deviation > 0·6 years

There is a greater spread of ages in the combined group than in a single year group.