DISCRETE MATHS (Q 6 & 7, PAPER 2)

2004

6 (a) A committee of five is to be selected from six students and three teachers.

(i) How many different committees of five are possible?

(ii) How many of these possible committees have three students and two teachers?

6 (b) (i) Solve the difference equation $3u_{n+2} - 2u_{n+1} - u_n = 0$, where $n \geq 0$, given that $u_0 = 3$ and $u_1 = 7$.

(ii) Evaluate $\lim_{n\to\infty} u_n$.

6 (c) Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black.
Four cards are selected at random from the eight cards.
Find the probability that the four cards selected are:

(i) all of different colours

(ii) two odd-numbered cards and two even-numbered cards

(iii) all of different colours, two odd-numbered and two even-numbered.

SOLUTION

6 (a) (i)

No. of ways you can pick five from nine people: ${}^{9}C_5 = 126$

6S 3T
↓
5 People

6 (a) (ii)

No. of ways you can pick three students from six students and two teachers from three teachers: ${}^{6}C_3 \times {}^{3}C_2 = 20 \times 3 = 60$

6S 3T
↓ ↓
3S 2T

6 (b) (i)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $3u_{n+2} - 2u_{n+1} - u_n = 0$

2. $3x^2 - 2x - 1 = 0$

3. $\therefore (3x+1)(x-1) = 0 \Rightarrow \alpha = 1,\ \beta = -\frac{1}{3}$

4. $u_n = l(1)^n + m(-\frac{1}{3})^n = l + m(-\frac{1}{3})^n$

5. $u_0 = 3 = l + m$ and $u_1 = 7 = l + m(-\frac{1}{3}) \Rightarrow 21 = 3l - m$

Solving these equations simultaneously $\Rightarrow l = 6,\ m = -3$

Ans: $u_n = 6 - 3(-\frac{1}{3})^n$

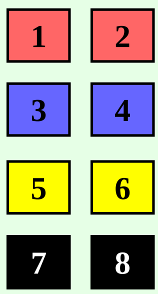
6 (b) (ii)

You may be asked to find a limit to infinity.

Remember $\lim_{n\to\infty} r^n = 0,\ -1 < r < 1$ but $\lim_{n\to\infty} r^n = \infty,\ r < -1,\ r > 1$

$\lim_{n\to\infty} u_n = 6$

6 (c) (i)



Find the probability of picking four different colours in a definite order and multiply this probability by the number of combinations of four colours.

p(Red, Blue, Yellow, Black) $= \frac{2}{8} \times \frac{2}{7} \times \frac{2}{6} \times \frac{2}{5} = \frac{1}{105}$

p(Four different colours) $= \frac{1}{105} \times 4! = \frac{8}{35}$

6 (c) (ii)

p(Odd, Odd, Even, Even) $= \frac{4}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{3}{5} = \frac{3}{35}$

p(Two odd-numbers and two even-numbers) $= \frac{3}{35} \times 6 = \frac{18}{35}$

6 (c) (iii)

p(Odd red, Odd blue, Even yellow, Even black) $= \frac{1}{8} \times \frac{1}{7} \times \frac{1}{6} \times \frac{1}{5} = \frac{1}{1680}$

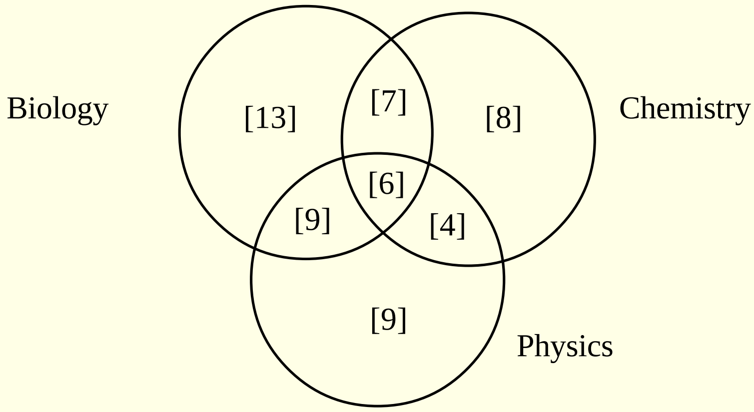
p(Four different colours, two odd-numbers and two even-numbers) $= \frac{1}{1680} \times 4! \times 6 = \frac{3}{35}$

7 (a) At the Olympic Games, eight lanes are marked on the running track. Each runner is allocated to a different lane. Find the number of ways in which the runners in a heat can be allocated to these lanes when there are

(i) eight runners in the heat

(ii) five runners in the heat and any five lanes may be used.

7 (b) In a class of 56 students, each studies at least one of the subjects Biology, Chemistry, Physics. The Venn diagram shows the number of students studying the various combinations of subjects.



(i) A student is picked at random from the whole class. Find the probability that the student does not study Biology.

(ii) A student is picked at random from those who study at least two of the subjects. Find the probability that the student does not study Biology.

(iii) Two students are picked at random from the whole class. Find the probability that they both study Physics.

(iv) Two students are picked at random from those who study Chemistry. Find the probability that exactly one of them studies Biology.

7 (c) The mean of the real numbers p, q and r is $\bar{x}$ and the standard deviation is σ.

(i) Show that the mean of the four numbers of p, q, r and $\bar{x}$ is also $\bar{x}$.

(ii) The standard deviation of p, q, r and $\bar{x}$ is k. Show that $k:\sigma = \sqrt{3}:2$.

Solution

7 (a) (i)

The first runner has a choice of 8 lanes, the second runner has a choice of 7 lanes once the first runner occupies a lane and so on.

$8\times7\times6\times5\times4\times3\times2\times1 = 40,320$

or $8! = 40,320$

or ${}^8P_8 = 40,320$

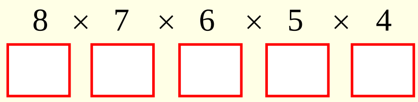
$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

7 (a) (ii)

The first runner has a choice of 8 lanes, the second runner has a choice of 7 lanes once the first runner occupies a lane and so on down to the fifth runner.

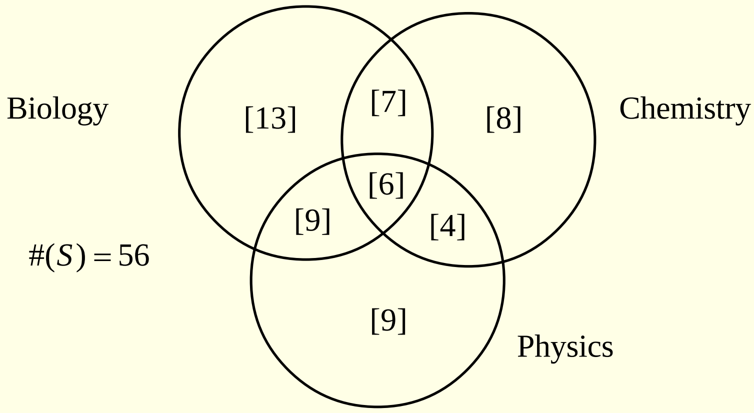
$8 \times 7 \times 6 \times 5 \times 4 = 6,720$

or ${}^{8}P_5 = 6,720$



7 (b)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$ 12



7 (b) (i)

No. of students not studying Biology = 8 + 4 + 9 = 21

p(Student does not study Biology) $= \frac{21}{56} = \frac{3}{8}$

7 (b) (ii)

No. of students studying at least two subjects = 7 + 9 + 4 + 6 = 26

No. of students not studying Biology from these students = 4

p(Student does not study Biology from those studying at least two subjects) $= \frac{4}{26} = \frac{2}{13}$

7 (b) (iii)

No. of students studying Physics = 9 + 9 + 6 + 4 = 28

p(Two students studying Physics) $= \frac{28}{56} \times \frac{27}{55} = \frac{27}{110}$

7 (b) (iv)

No. of students studying Chemistry = 8 + 7 + 4 + 6 = 25

No. of these students studying Biology = 7 + 6 = 13

No. of these students not studying Biology = 8 + 4 = 12

p(One student studies Biology and one does not study Biology) $= \frac{13}{25} \times \frac{12}{24} \times 2 = \frac{13}{25}$

7 (c) (i)

$$\bar{x} = \frac{x_1 + x_2 + \dots\dots\dots\dots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$ 3

Mean of three numbers $\bar{x} = \dfrac{p+q+r}{3}$

Mean of four numbers $= \dfrac{p+q+r+\bar{x}}{4} = \dfrac{p+q+r+(\frac{p+q+r}{3})}{4} \times \dfrac{3}{3} = \dfrac{3p+3q+3r+p+q+r}{12}$

$= \dfrac{4p+4q+4r}{12} = \dfrac{p+q+r}{3} = \bar{x}$

7 (c) (ii)

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ 6

where deviation $d = (x - \bar{x}) = (\text{Number} - \text{Mean})$

x	d	d^2
p	$p-\bar{x}$	$(p-\bar{x})^2$
q	$q-\bar{x}$	$(q-\bar{x})^2$
r	$r-\bar{x}$	$(r-\bar{x})^2$

$$\sigma = \sqrt{\frac{\sum d^2}{N}} = \sqrt{\frac{(p-\bar{x})^2+(q-\bar{x})^2+(r-\bar{x})^2}{3}}$$

x	d	d^2
p	$p-\bar{x}$	$(p-\bar{x})^2$
q	$q-\bar{x}$	$(q-\bar{x})^2$
r	$r-\bar{x}$	$(r-\bar{x})^2$
$\bar{x}$	$\bar{x}-\bar{x}$	0

$$k = \sqrt{\frac{\sum d^2}{N}} = \sqrt{\frac{(p-\bar{x})^2+(q-\bar{x})^2+(r-\bar{x})^2+0}{4}}$$

$$\frac{k}{\sigma} = \frac{\sqrt{\dfrac{(p-\bar{x})^2+(q-\bar{x})^2+(r-\bar{x})^2}{4}}}{\sqrt{\dfrac{(p-\bar{x})^2+(q-\bar{x})^2+(r-\bar{x})^2}{3}}} = \frac{\sqrt{3}}{2} \Rightarrow k:\sigma = \sqrt{3}:2$$