Discrete Maths (Q 6 & 7, Paper 2)

2003

6 (a) Eight people, including Kieran and Anne, are available to form a committee. Five people must be chosen for the committee.

(i) In how many ways can the committee be formed if both Kieran and Anne must be chosen?

(ii) In how many ways can the committee be formed if neither Kieran nor Anne can be chosen?

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} + 3u_n = 0,$ where $n \geq 0,$ given that $u_o = -2$ and $u_1 = 4.$

(ii) Verify that the solution you have obtained in (**i**) satisfies the difference equation.

6 (c) Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.

(i) What is the probability that the disc with the number 7 is drawn?

(ii) What is the probability that the three numbers on the discs drawn are odd?

(iii) What is the probability that the product of the three numbers on the discs drawn is even?

(iv) What is the probability that the smallest number on the discs drawn is 4?

Solution

6 (a) (i)

If both Kieran and Anne must be chosen, then the number of ways of choosing three people from the remaining six people is ${}^6C_3 = 20.$

6 (a) (ii)

If neither Kieran nor Anne are to be chosen, then the number of ways of choosing five people from six people is ${}^6C_5 = 6.$

6 (b) (i)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS
1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $u_{n+2} - 4u_{n+1} + 3u_n = 0$

2. $x^2 - 4x + 3 = 0$

3. $(x-1)(x-3) = 0 \Rightarrow \alpha = 1, \ \beta = 3$

4. $u_n = l(1)^n + m(3)^n = l + m(3)^n$

5. $u_0 = -2 = l + m$ and $u_1 = 4 = l + 3m$

Solving these equations simultaneously $\Rightarrow l = -5, \ m = 3$

Ans: $u_n = 3(3)^n - 5$

6 (b) (ii)

$u_n = 3(3)^n - 5$

$u_{n+1} = 3(3)^{n+1} - 5$

$u_{n+2} = 3(3)^{n+2} - 5$

$\Rightarrow 3(3)^{n+2} - 5 - 4[3(3)^{n+1} - 5] + 3[3(3)^n - 5] = 3(3)^{n+2} - 5 - 12(3)^{n+1} + 20 + 9(3)^n - 15$

$= (3)^n[3(3)^2 - 12(3) + 9] = (3)^n[27 - 36 + 9] = (3)^n[0] = 0$

6 (c)

1	2	3	4	5	6	7	8	9	10

6 (c) (i)

Find the probability of a particular combination and then calculate the number of ways in which that combination can take place.

p(7, Not 7, Not 7) $= \frac{1}{10} \times \frac{9}{9} \times \frac{8}{8} = \frac{1}{10}$

7	N7	N7

No. of combinations $= \frac{3!}{2!} = 3$

p(7 is drawn) $= \frac{1}{10} \times 3 = \frac{3}{10}$

6 (c) (ii)

p(Odd, Odd, Odd) $= \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{12}$

6 (c) (iii)

The product of three odd numbers is odd. Every other combination of three numbers is even.

p(Product is even) = 1 – p(Product is odd) $= 1 - \frac{1}{12} = \frac{11}{12}$

6 (c) (iv)

$p(4, >4, >4) = \frac{1}{10} \times \frac{6}{9} \times \frac{5}{8} = \frac{1}{24}$

4 **>4** **>4**

No. of combinations $= \frac{3!}{2!} = 3$

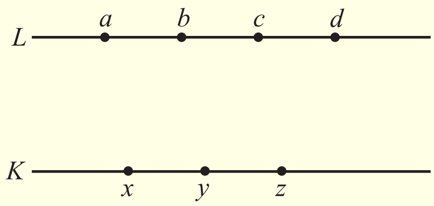
p(Smallest number drawn is 4) $= \frac{1}{24} \times 3 = \frac{1}{8}$

7 (a) Five cars enter a car park. There are exactly five vacant spaces in the car park.

(i) In how many different ways can the five cars park in the vacant spaces?

(ii) Two of the cars leave the car park without parking. In how many different ways can the remaining three cars park in the five vacant spaces?

7 (b)



L and K are distinct parallel lines.

a, b, c and d are points on L such that $|ab| = |bc| = |cd| = 1$ cm.

x, y and z are points on K such that $|xy| = |yz| = 1$ cm.

(i) How many different triangles can be constructed using three of the named points as vertices?

(ii) How many different quadrilaterals can be constructed using four of the named points as vertices?

(iii) How many different parallelograms can be constructed using four of the named points as vertices?

(iv) If one quadrilateral is constructed at random, what is the probability that it is *not* a parallelogram?

7 (c) The mean of the real numbers a and b is $\bar{x}$. The standard deviation is σ.

(i) Express σ in terms of a, b and $\bar{x}$.

(ii) Hence, express σ in terms of a and b only.

(iii) Show that $\bar{x}^2 - \sigma^2 = ab$.

SOLUTION

7 (a) (i)

The first car has a choice of 5 spaces, the second car has a choice of 4 spaces once the first car occupies a space and so on down to the fifth car.

$5 \times 4 \times 3 \times 2 \times 1 = 120$

or ${}^5P_5 = 120$

or $5! = 120$

$5 \times 4 \times 3 \times 2 \times 1$

7 (a) (ii)

The first car has a choice of 5 spaces, the second car has a choice of 4 spaces once the first car occupies a space and the third car has a choice of 3 spaces once the first two cars occupy their spaces.

$5 \times 4 \times 3 = 60$

or ${}^5P_3 = 60$

$5 \times 4 \times 3$

7 (b) (i)

To calculate the number of triangles you need to pick any two points from L $({}^4C_2)$ AND one point from K $({}^3C_1)$ OR one point from L $({}^3C_2)$ AND two points from K $({}^4C_1)$.

No. of triangles = ${}^4C_2 \times {}^3C_1 + {}^3C_2 \times {}^4C_1 = 6 \times 3 + 3 \times 4 = 18 + 12 = 30$

7 (b) (ii)

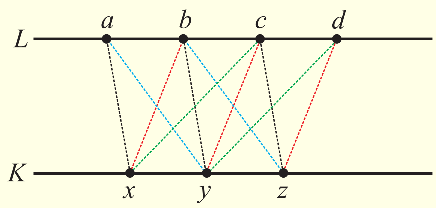
To calculate the number of quadrilaterals you need to pick any two points from L $({}^4C_2)$ AND any two points from K $({}^3C_2)$.

No. of quadrilaterals = ${}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

7 (b) (iii)

To calculate the number of parallelograms you need to draw them.

No. of parallelograms = 8



7 (b) (iv)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$ 12

#S: No. of quadrilaterals = 18

#E: No. of quadrilaterals that are *not* parallelograms = 10

p(Quadrilateral that is not a parallelogram) $= \frac{10}{18} = \frac{5}{9}$

7 (c) (i)

$$\overline{x} = \frac{x_1 + x_2 + \dots\dots\dots\dots + x_N}{N} = \frac{\text{Sum of the Numbers}}{\text{Number of Numbers}} = \frac{\sum x}{N}$$ 3

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ 6

where deviation $d = (x - \overline{x}) = (\text{Number} - \text{Mean})$

x	d	d^2
a	$a - \overline{x}$	$(a - \overline{x})^2$
b	$b - \overline{x}$	$(b - \overline{x})^2$

$$\sigma = \sqrt{\frac{(a - \overline{x})^2 + (b - \overline{x})^2}{2}}$$

7 (c) (ii)

$$\overline{x} = \frac{a+b}{2} \Rightarrow \sigma = \sqrt{\frac{(a - \frac{a+b}{2})^2 + (b - \frac{a+b}{2})^2}{2}} = \sqrt{\frac{(\frac{a-b}{2})^2 + (\frac{b-a}{2})^2}{2}}$$

$$= \sqrt{\frac{(a-b)^2 + (b-a)^2}{8}} = \sqrt{\frac{2(a-b)^2}{8}} = \sqrt{\frac{(a-b)^2}{4}} = \frac{a-b}{2}$$

7 (c) (iii)

$$\overline{x}^2 - \sigma^2 = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = \frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}{4} = \frac{4ab}{4} = ab$$