DISCRETE MATHS (Q 6 & 7, PAPER 2)

2002

6 (a) Nine friends wish to travel in a car. Only two of them, John and Mary, have licences to drive. Only five people can fit in the car (i.e. the driver and four others).
In how many ways can the group of five people be selected if

(i) both John and Mary are included

(ii) either John or Mary is included, but not both?

Later, another one of the nine friends, Anne, gets a driving licence.

(iii) The next time the journey is made, in how many ways can the group of five be chosen, given that at least one licenced driver must be included?

6 (b) (i) Solve the difference equation $6u_{n+2} - 5u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 5$ and $u_1 = 2$.

(ii) Find an expression in n for the sum of the terms $u_0 + u_1 + u_2 + ... + u_n$.
(Hint: It is the sum of two geometric series.)

(iii) Evaluate the sum to infinity of this series (that is: $\sum_{n=0}^{\infty} u_n$).

SOLUTION

6 (a) (i)

If John and Mary are included, you must calculate the number of ways you can pick three people from seven people.

$^7C_3 = 35$

6 (a) (ii)

If John is included and Mary is not, you must calculate the number of ways you can pick four people from seven people *OR* if Mary is included and John is not, you must also calculate the number of ways you can pick four people from seven people.

$2 \times {}^7C_4 = 70$

6 (a) (iii)

No. of combinations with at least one qualified driver present
= Total no. of combinations – No. of combinations with no qualified driver

$= {}^9C_5 - {}^6C_5 = 126 - 6 = 120$

6 (b) (i)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find *l*, *m* using extra conditions (boundary conditions).

1. $6u_{n+2} - 5u_{n+1} + u_n = 0$

2. $6x^2 - 5x + 1 = 0$

3. $(2x-1)(3x-1) = 0 \Rightarrow \alpha = \frac{1}{2},\ \beta = \frac{1}{3}$

4. $u_n = l(\frac{1}{2})^n + m(\frac{1}{3})^n$

5. $u_0 = 5 = l + m$ and $u_1 = 2 = \frac{1}{2}l + \frac{1}{3}m \Rightarrow 12 = 3l + 2m$

Solving these equations simultaneously $\Rightarrow l = 2,\ m = 3$

Ans: $u_n = 2(\frac{1}{2})^n + 3(\frac{1}{3})^n$

6 (b) (ii)

Summing formula: $S_n = \dfrac{a(1-r^n)}{(1-r)}$ **5**

$$u_n = 2(\tfrac{1}{2})^n + 3(\tfrac{1}{3})^n = 2[1 + (\tfrac{1}{2}) + (\tfrac{1}{2})^2 + \ldots + (\tfrac{1}{2})^{n+1}] + 3[1 + (\tfrac{1}{3}) + (\tfrac{1}{3})^2 + \ldots + (\tfrac{1}{3})^{n+1}]$$

↑ $a = 1,\ r = \frac{1}{2},\ S_{n+1}$ ↑ $a = 1,\ r = \frac{1}{3},\ S_{n+1}$

$$\Rightarrow u_n = \frac{2(1-(\frac{1}{2})^{n+1})}{1-\frac{1}{2}} + \frac{3(1-(\frac{1}{3})^{n+1})}{1-\frac{1}{3}} = 4(1-(\tfrac{1}{2})^{n+1}) + \tfrac{9}{2}(1-(\tfrac{1}{3})^{n+1})$$

6 (b) (iii)

$$\sum_{n=0}^{\infty} u_n = 4(1-0) + \tfrac{9}{2}(1-0) = \tfrac{17}{2}$$

FAMOUS LIMITS

$\lim_{n\to\infty} r^n = 0$ for $-1 < r < 1$. **Example**: $\lim_{n\to\infty} (\frac{3}{5})^n = 0$

$\lim_{n\to\infty} r^n = \infty$ for $r > 1,\ r < -1$. **Example**: $\lim_{n\to\infty} (\frac{3}{2})^n = \infty$

7 (a) Two unbiased dice, each with faces numbered 1 to 6, are thrown.

(i) What is the probability of getting a total equal to 8?

(ii) What is the probability of getting a total less than 8?

7 (b) The table below shows the prices of various commodities in the year 2000, as a percentage of their prices in 1999. These are called *price relatives*. (For example, the price relative for *Food, Drink & Other Goods* is 105, indicating that the cost of these items was 5% greater in 2000 than in 1999.)

The table also shows the weight assigned to each commodity. The weight represents the importance of the commodity to the average consumer.

Commodity	Weight	Price in 2000 as % of price in 1999
Housing	8	110
Fuel and Transport	19	108
Tobacco	5	116
Services	16	105
Clothing & Durable Goods	10	97
Food, Drink & Other Goods	42	105

(i) Calculate the weighted mean of the price relatives in the table.

(ii) Calculate, correct to two decimal places, the change in the weighted mean if *Tobacco* is removed from consideration.

7 (c) A palindromic number is one that reads the same backwards as forwards, such as 727 or 38183.

(i) The year, 2002, is a palindromic year. When is the next palindromic year?

(ii) How many palindromic years are there from 1000 to 9999 inclusive?

(iii) A whole number, greater than 9 and less than 10 000, is selected at random. What is the probability that the number is palindromic?

Solution

7 (a)

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$ 12

The table above is the sample space, S. $\Rightarrow \#(S) = 36$

7 (a) (i)

$E = \{(6, 2), (5, 3), (4, 4), (3, 5), (2, 6)\}$

$\Rightarrow \#(E) = 5 \Rightarrow p = \frac{5}{36}$

7 (a) (ii)

$\#(E) = 21 \Rightarrow p = \frac{21}{36} = \frac{7}{12}$

7 (b) (i)

WEIGHTED MEAN $\overline{w}$ $\quad \overline{w} = \frac{\sum wx}{\sum w}$ 5

Commodity	x	w	wx
Housing	110	8	880
Fuel and Transport	108	19	2052
Tobacco	116	5	580
Services	105	16	1680
Clothing & Durable Goods	97	10	970
Food, Drink & Other Goods	105	42	4410
		100	10572

$$\overline{w} = \frac{\sum wx}{\sum w} = \frac{10572}{100} = 105 \cdot 72$$

7 (b) (ii)

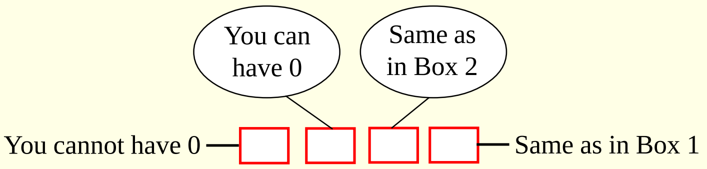
Commodity	x	w	wx
Housing	110	8	880
Fuel and Transport	108	19	2052
Services	105	16	1680
Clothing & Durable Goods	97	10	970
Food, Drink & Other Goods	105	42	4410
		95	9992

New weighted mean: $\overline{w}_{new} = \frac{\sum wx}{\sum w} = \frac{9992}{95} = 105 \cdot 18$

Change in weighted mean = $105 \cdot 72 - 105 \cdot 18 = 0 \cdot 54$

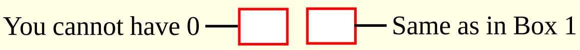
7 (c) (i)
The next palindromic year is 2112.

7 (c) (ii)
You are being asked how many four digit palindromic numbers there are.



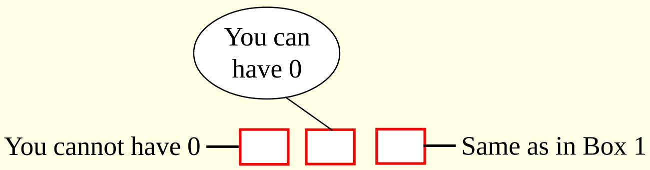
Number of ways = 9 × 10 × 1 × 1 = 90

7 (c) (iii)
Firstly, you are being asked how many two digit, three digit and four digit palindromic numbers exist?
2 digit palindromic numbers:



Number of ways = 9 × 1 = 9

3 digit palindromic numbers:



Number of ways = 9 × 10 × 1 = 90

No. of palindromic numbers between 9 and 10,000 = 9 + 90 + 90 = 189
No. of numbers between 9 and 10,000 = 9,990

$$p(E) = \frac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \frac{\#(E)}{\#(S)}$$ 12

$p(\text{Picking a palindromic number}) = \frac{189}{9990} = \frac{7}{370}$