DISCRETE MATHS (Q 6 & 7, PAPER 2)

2001

6 (a) (i) How many different sets of three books or of four books can be selected from six different books?

(ii) How many of the above sets contain one particular book?

6 (b) Solve the difference equation $u_{n+2} - 8u_{n+1} + 11u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2\sqrt{15}$.

6 (c) A box contains four silver coins, two gold coins and x copper coins. Two coins are picked at random, and without replacement, from the box.

(i) Write down an expression in x for the probability that the two coins are copper.

If it is known that the probability of picking two copper coins is $\frac{4}{13}$,

(ii) how many coins are in the box and

(iii) what is the probability that neither of the two coins picked is copper?

SOLUTION

6 (a) (i)

You need to choose sets of three books from six books *OR* sets of four books from six books.

$^6C_3 + {}^6C_4 = 20 + 15 = 35$

6 (a) (ii)

If a particular book is included, this means you must choose sets of two books from five books *OR* sets of three books from five books.

$^5C_2 + {}^5C_3 = 10 + 10 = 20$

6 (b)

$u_n = l(\alpha)^n + m(\beta)^n$ **1**

STEPS

1. Write the Second Order Difference Equation in decreasing order of subscripts: $pu_{n+2} + qu_{n+1} + ru_n = 0$
2. Write down the corresponding quadratic equation: $px^2 + qx + r = 0$
3. Solve this equation to find α, β.
4. Write solution as: $u_n = l(\alpha)^n + m(\beta)^n$
5. Find l, m using extra conditions (boundary conditions).

1. $u_{n+2} - 8u_{n+1} + 11u_n = 0$

2. $x^2 - 8x + 11 = 0$

3. $x = \dfrac{8 \pm \sqrt{64 - 4(1)(11)}}{2} = \dfrac{8 \pm \sqrt{20}}{2} = \dfrac{8 \pm 2\sqrt{5}}{2} = 4 \pm \sqrt{5}$

$\Rightarrow \alpha = 4 + \sqrt{5},\ \beta = 4 - \sqrt{5}$

4. $u_n = l(4 + \sqrt{5})^n + m(4 - \sqrt{5})^n$

5. $u_0 = l + m = 0 \Rightarrow m = -l$ and $u_1 = l(4 + \sqrt{5}) + m(4 - \sqrt{5}) = 2\sqrt{15}$

Solving these equations simultaneously:

$l(4 + \sqrt{5}) - l(4 - \sqrt{5}) = 2\sqrt{15} \Rightarrow 2l\sqrt{5} = 2\sqrt{15}$

$\Rightarrow l = \sqrt{3},\ m = -\sqrt{3}$

Ans: $u_n = \sqrt{3}(4 + \sqrt{5})^n - \sqrt{3}(4 - \sqrt{5})^n$

6 (c)

4 Silver coins
2 Gold coins
x Copper coins

$p(E) = \dfrac{\text{Number of desired outcomes}}{\text{Total possible number of outcomes}} = \dfrac{\#(E)}{\#(S)}$ **12**

6 (c) (i)

$p(\text{Copper, Copper}) = \dfrac{x}{(x+6)} \times \dfrac{(x-1)}{(x+5)}$

6 (c) (ii)

$\dfrac{x}{(x+6)} \times \dfrac{(x-1)}{(x+5)} = \dfrac{4}{13} \Rightarrow 13x(x-1) = 4(x+6)(x+5)$

$\Rightarrow 13x^2 - 13x = 4x^2 + 44x + 120 \Rightarrow 9x^2 - 57x - 120 = 0$

$\Rightarrow 3x^2 - 19x - 40 = 0 \Rightarrow (3x+5)(x-8) = 0$

$\Rightarrow x = -\frac{5}{3},\ 8$

There are 8 copper coins (this number has to be a natural number.)
Total number of coins = 14

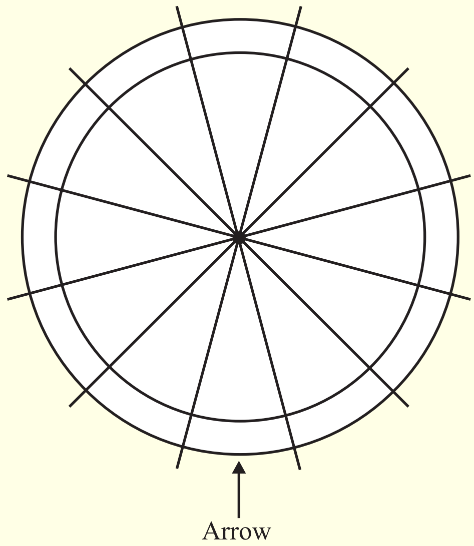
6 (c) (iii)

$p(\text{Not Copper, Not Copper}) = \frac{6}{14} \times \frac{5}{13} = \frac{15}{91}$

7 (a) (i) In how many different ways can four of the letters of the word FRIDAY be arranged if each letter is used no more than once in each arrangement?

(ii) How many of the above arrangements begin with the letter D and end with a vowel?

7 (b) To play a game a player spins a wheel. The wheel is fixed to a wall. It spins freely around its centre point. Its rim is divided equally into twelve regions. Three of the regions are coloured red. Four are coloured blue. Five are coloured green. When the wheel stops an arrow fixed to the wall points to one of the regions. All the regions are equally likely to stop at the arrow. The colour of this region is the outcome of the game. When the game is played twice, calculate the probability that



(i) both outcomes are green
(ii) both outcomes are the same colour
(iii) the first outcome is red and the second is green
(iv) one outcome is green and the other is blue.

7 (c) Consider the numbers $1, k, 3k-2, 9$ where $k \in \mathbf{Z}$. The mean of these numbers is $\bar{x}$. The standard deviation is σ.

(i) Express $\bar{x}$ in terms of k.

(ii) Given that $\sigma = \sqrt{20}$, find the value of k.

Solution

7 (a) (i)

There are six ways to fill the first box. Once, this is filled there are five ways to fill the second box and so on.

$6 \times 5 \times 4 \times 3 = 360$

[ ] [ ] [ ] [ ]

OR

The number of ways of arranging four letters from six letters is ${}^6P_4 = 360$

7 (a) (ii)

$1 \times 4 \times 3 \times 1 = 12$

[**D**] [ ] [ ] [**A**]

$\oplus$

OR

$1 \times 4 \times 3 \times 1 = 12$

[**D**] [ ] [ ] [**I**]

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There are two possibilities for ending in a vowel. Consider each one separately and add together the two answers. First possibility: There is only one way to fill the first box (with a **D**) and one way to fill the last box (with an **A**). This means there are four ways to fill the second box and three ways to fill the third box. There are 12 arrangements. Similarly there are 12 arrangement for the second possibility giving a total of 24 arrangements.

7 (b)

3 Red 4 Blue 5 Green

This problem could also be viewed as a container with 12 discs where the discs are picked with replacement.

7 (b) (i)

$p(\text{Green and Green}) = \frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$

7 (b) (ii)

$p(\text{Green and Green})$ OR $p(\text{Red and Red})$ OR $p(\text{Blue and Blue})$

$= \frac{5}{12} \times \frac{5}{12} + \frac{3}{12} \times \frac{3}{12} + \frac{4}{12} \times \frac{4}{12} = \frac{25}{144} + \frac{9}{144} + \frac{16}{144} = \frac{50}{144} = \frac{25}{72}$

7 (b) (iii)

$p(\text{Red AND THEN Green}) = \frac{3}{12} \times \frac{5}{12} = \frac{15}{144} = \frac{5}{48}$

7 (b) (iv)

$p(\text{Green and Blue}) = \frac{5}{12} \times \frac{4}{12} \times 2 = \frac{5}{18}$

7 (c) (i)

WEIGHTED MEAN $\overline{w}$ $\quad \overline{w} = \frac{\sum wx}{\sum w}$ **5**

$$\overline{x} = \frac{1 + k + 3k - 2 + 9}{4} = \frac{4k + 8}{4} = k + 2$$

7 (c) (ii)

$$\sigma = \sqrt{\frac{\text{Sum of (Deviations)}^2}{\text{Number of numbers}}} = \sqrt{\frac{\sum d^2}{N}}$$ **6**

where deviation $d = (x - \overline{x}) = (\text{Number} - \text{Mean})$

x	d	d^2
1	$k+1$	$(k+1)^2$
k	2	4
$3k-2$	$2k-4$	$(2k-4)^2$
9	$k-7$	$(k-7)^2$

$$\sigma = \sqrt{\frac{(k+1)^2 + 4 + (2k-4)^2 + (k-7)^2}{4}} = \sqrt{20}$$

$\Rightarrow k^2 + 2k + 1 + 4 + 4k^2 - 16k + 16 + k^2 - 14k + 49 = 80$

$\Rightarrow 6k^2 - 28k - 10 = 0 \Rightarrow 3k^2 - 14k - 5 = 0$

$\Rightarrow (3k+1)(k-5) = 0 \Rightarrow k = 5 \ (k \in \mathbf{Z})$