

DISCRETE MATHS (Q 6 & 7, PAPER 2)

LESSON NO. 1: DIFFERENCE EQUATIONS

2006

6 (b) (i) Solve the difference equation $6u_{n+2} - 7u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 8$ and $u_1 = 3$.

(ii) Verify that the solution to part (i) also satisfies the difference equation $6u_{n+1} - u_n - 10 = 0$.

2005

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} - 8u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2$.

(ii) Verify that your solution gives the correct value for u_2 .

2004

6 (b) (i) Solve the difference equation $3u_{n+2} - 2u_{n+1} - u_n = 0$, where $n \geq 0$, given that $u_0 = 3$ and $u_1 = 7$.

(ii) Evaluate $\lim_{n \rightarrow \infty} u_n$.

2003

6 (b) (i) Solve the difference equation $u_{n+2} - 4u_{n+1} + 3u_n = 0$, where $n \geq 0$, given that $u_0 = -2$ and $u_1 = 4$.

(ii) Verify that the solution you have obtained in (i) satisfies the difference equation.

2002

6 (b) (i) Solve the difference equation $6u_{n+2} - 5u_{n+1} + u_n = 0$, where $n \geq 0$, given that $u_0 = 5$ and $u_1 = 2$.

(ii) Find an expression in n for the sum of the terms $u_0 + u_1 + u_2 + \dots + u_n$.
(Hint: It is the sum of two geometric series.)

(iii) Evaluate the sum to infinity of this series (that is: $\sum_{n=0}^{\infty} u_n$).

2001

6 (b) Solve the difference equation $u_{n+2} - 8u_{n+1} + 16u_n = 0$, where $n \geq 0$, given that $u_0 = 0$ and $u_1 = 2\sqrt{15}$.

ANSWERS

2006 6 (b) (i) $2 + (\frac{1}{6})^{n-1}$

2005 6 (b) (i) $u_n = \frac{\sqrt{3}}{6} (2 + 2\sqrt{3})^n - \frac{\sqrt{3}}{6} (2 - 2\sqrt{3})^n$ (ii) $u_2 = 8$

2004 6 (b) (i) $u_n = 6 - 3(-\frac{1}{3})^n$ (ii) 6

2003 6 (b) (i) $u_n = 3(3)^n - 5$

2004 6 (b) (i) $u_n = 2(\frac{1}{2})^n + 3(\frac{1}{3})^n$ (ii) $4[1 - \frac{1}{2^{n+1}}] + \frac{9}{2}[1 - \frac{1}{3^{n+1}}]$ (iii) $8\frac{1}{2}$

2001 6 (b) $u_n = \sqrt{3}(4 + \sqrt{5})^n - \sqrt{3}(4 - \sqrt{5})^n$