DISCRETE MATHS (Q 6 & 7, PAPER 2)

2000

6 (a) A bank gives each of its customers a four digit personal identification number which is formed from the digits 0 to 9 inclusive. Examples are 2475, 0865 and 3422.

(i) How many different personal identification numbers can the bank use?

(ii) If the bank decides not to use personal identification numbers that begin with 0, how many different numbers can it then use?

6 (b) (i) Solve the difference equation $12u_{n+2} - 8u_{n+1} + u_n = 0,$ where $n \geq 0,$ given that $u_0 = \frac{1}{15}$ and $u_1 = \frac{7}{30}$.

(ii) Hence, express u_3 in the form $\frac{p}{q}$ where $p, q \in \mathbf{N}$.

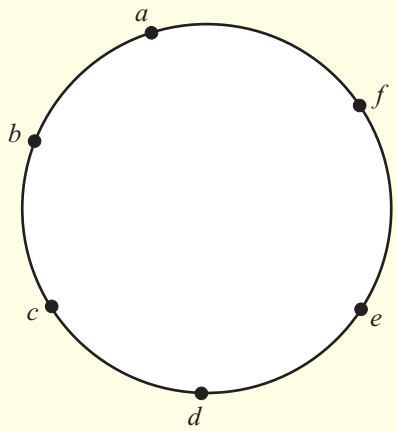
6 (c) Six red discs, numbered from 1 to 6, and four green discs, numbered from 7 to 10, are placed in box A. Ten blue discs, numbered from 1 to 10, are placed in box B. Two discs are drawn from box A and discs are drawn from box B. The four discs are drawn at random and without replacement.
Find the probability that the discs drawn are

(i) two red discs and two even numbered blue discs

(ii) one red disc, one green disc and two blue discs with all fours discs odd numbered

(iii) one red disc, one green disc and two blue discs with the total on the red and green discs equal to 10 and the total on the blue discs also equal to 10.

7 (a) The points a, b, c, d, e and f lie on a circle.



(i) If these points are used as vertices, how many different quadrilateral can be formed?

(ii) How many of these quadrilaterals will have $[ab]$ as one side?

7 (b) Three cards are drawn, at random and without replacement, from a pack of 52 playing cards. Find the probability that

(i) the three cards drawn are the Jack of clubs, the Queen of clubs and the King of clubs

(ii) the three cards are aces

(iii) two cards are black and one card is a diamond

(iv) the three cards are of the same colour.

7 (c) The mean of the real numbers q, r, s and t is $\overline{x}$ and the standard deviation is σ. Consider the numbers $\beta q+\alpha$, $\beta r+\alpha$, $\beta s+\alpha$ and $\beta t+\alpha$ where $\beta, \alpha \in \mathbf{R}$ and $\beta > 0$.

(i) Show that the mean of these numbers is $\beta\overline{x}+\alpha$.

(ii) Show that the standard deviation of these numbers is $\beta\sigma$.

ANSWERS

6 (a) (i) 10,000 (ii) 9,000

6 (b) (i) $u_n = \frac{2}{3}(\frac{1}{2})^n - \frac{3}{5}(\frac{1}{6})^n$ (ii) $\frac{29}{360}$

6 (c) (i) $\frac{2}{27}$ (ii) $\frac{4}{135}$ (iii) $\frac{4}{675}$

7 (a) (i) 15 (ii) 6

7 (b) (i) $\frac{1}{22{,}100}$ (ii) $\frac{1}{5{,}525}$ (iii) $\frac{13}{68}$ (iv) $\frac{4}{17}$