Calculus Option (Q 8, Paper 2)

Lesson No. 1: Integration by Parts

2005

8 (a) Use integration by parts to find $\int x^2 \ln x \, dx$.

Solution

8 (a)

Parts formula $\int u\,dv = uv - \int v\,du$ **1**

Steps

1. Call the original integral I (ignore limits of integration).
2. Let u equal the higher function in the list and find du by differentiation; Let dv equal what is left and find v by integration. **Note**: **LIATE** helps you to remember the order.
3. Substitute into Parts Formula. You will now be left with $\int v\,du$. You will either be able to integrate this integral normally or you must integrate by parts again.
4. If there are limits of integration, do them at the end.

List of Functions

1. **L**og
2. **I**nverse Trig
3. **A**lgebraic
4. **T**rigonometry
5. **E**xponential

1. $I = \int x^2 \ln x \, dx$

2.

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{3} x^3$

3. $I = uv - \int v\,du = (\ln x)(\frac{1}{3}x^3) - \int \frac{1}{3}x^3(\frac{1}{x})\,dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$

$\therefore I = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$

2004

8 (a) Use integration by parts to find $\int x \sin x \, dx$.

Solution

8 (a)

1. $I = \int x \sin x \, dx$

2.

$u = x \qquad dv = \sin x \, dx$

$du = 1 \, dx \qquad v = -\cos x$

3. $I = uv - \int v du = -x\cos x - \int(-\cos x)\,dx = -x\cos x + \sin x + c$

2003

8 (a) Use integration by parts to find $\int xe^{-5x}dx$.

Solution

8 (a)

1. $I=\int xe^{-5x}\,dx$

2.

$u=x$	$dv=e^{-5x}dx$
$du=1\,dx$	$v=-\frac{1}{5}e^{-5x}$

3. $I=x(-\frac{1}{5}e^{-5x})-\int(-\frac{1}{5}e^{-5x})dx=-\frac{1}{5}xe^{-5x}+\frac{1}{5}\int e^{-5x}dx$

$\therefore I=-\frac{1}{5}xe^{-5x}-\frac{1}{25}e^{-5x}+c$

2002

8 (a) Use integration by parts to find $\int x\ln x\,dx$.

Solution

8 (a)

1. $I=\int x\ln x\,dx$

2.

$u=\ln x$	$dv=x\,dx$
$du=\frac{1}{x}dx$	$v=\frac{1}{2}x^2$

3. $I=uv-\int v\,du=(\ln x)\frac{1}{2}x^2-\int(\frac{1}{2}x^2)(\frac{1}{x})\,dx=\frac{1}{2}x^2\ln x-\frac{1}{2}\int x\,dx$

$\therefore I=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+c$

2001

8 (a) Use integration by parts to find $\int x\cos x\,dx$.

Solution

8 (a)

1. $I=\int x\cos x\,dx$

2.

$u=x$	$dv=\cos x\,dx$
$du=1\,dx$	$v=\sin x$

3. $I=uv-\int v\,du\Rightarrow I=x\sin x-\int\sin x\,dx$

$\therefore I=x\sin x+\cos x+c$