Calculus Option (Q 8, Paper 2)

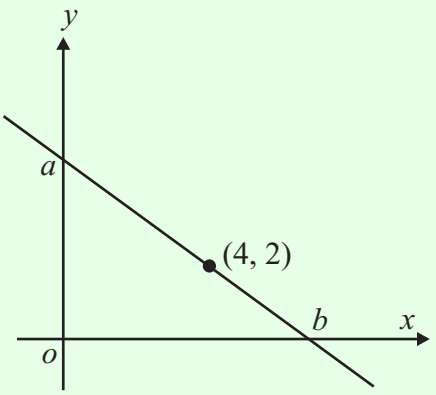
Lesson No. 4: Maximising and Minimising Functions

2006

8 (b) A line passes through the point (4, 2) and has slope m, where $m < 0$. The line intersects the axes at the points a and b.

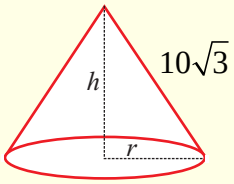
(i) Find the co-ordinates of a and b, in terms of m.

(ii) Hence, find the value of m for which the area of triangle aob is a minimum.



2005

8 (c) A cone has radius r cm, vertical height h cm and slant height $10\sqrt{3}$ cm. Find the value of h for which the volume is a maximum.



2004

8 (c) A solid cylinder has height h and radius r. The height of the cylinder, added to the circumference of its base, is equal to 3 metres.

(i) Express the volume of the cylinder in terms of r and π.

(ii) Find the maximum possible volume of the cylinder in terms of π.

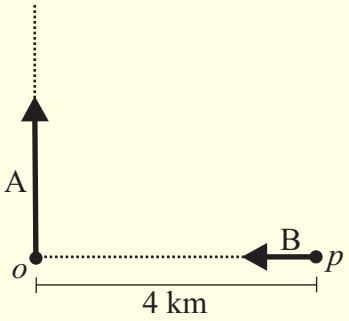
2003

8 (c) The point p is 4 km due east of the point o. At noon, A leaves o and travels north at a steady speed of 12 km/h. At the same time, B leaves p and travels towards o at a steady speed of 6 km/h.

(i) Write down expressions in x for the distances that A and B will each have travelled at x minutes after noon.

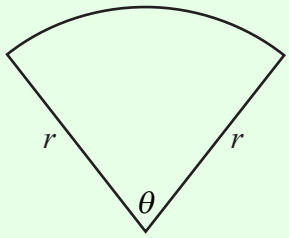
(ii) Find an expression in x for the distance that B will be from A at x minutes after noon.

(iii) At how many minutes after noon will B be closest to A?



2002

8 (b) The perimeter of a sector of a circle of radius r is 8 metres.



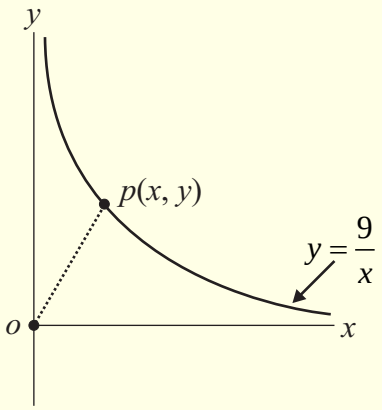
(i) Express θ in terms of r, where θ is the angle of the sector in radians as shown.

(ii) Hence, show that the area of the sector, in square metres, is $4r - r^2$.

(iii) Find the maximum possible area of the sector.

2001

8 (c) o is the origin, (0, 0). $p(x, y)$ is a point on the curve $y = \dfrac{9}{x}$, where $x > 0$. $|op|$ is the distance from the origin to p.



(i) Express $|op|$ in terms of x.

(ii) Given that there is one value of x for which $|op|$ is a minimum, find this value of x.

(iii) Hence, find the minimum value of $|op|$.

ANSWERS

2006 8 (b) (i) $a(0,\ 2-4m)$, $b(4-\frac{2}{m}, 0)$ (ii) $m = -\frac{1}{2}$

2005 8 (c) h = 10 cm

2004 8 (c) (i) $V = 3\pi r^2 - 2\pi^2 r^3$ (ii) $\dfrac{1}{\pi}$

2003 8 (c) (i) A: $\frac{x}{5}$ km; B: $\frac{x}{10}$ km (ii) $\sqrt{(\frac{x}{5})^2 + (4-\frac{x}{10})^2}$
(iii) 8 minutes

2002 8 (b) (i) $\frac{8}{r} - r$ (iii) 4

2001 8 (c) (i) $|op| = \sqrt{x^2 + \dfrac{81}{x^2}}$ (ii) x = 3 (iii) $3\sqrt{2}$