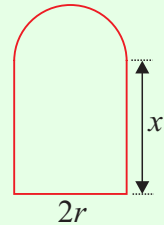


CALCULUS OPTION (Q 8, PAPER 2)

2011

8. (a) Use integration by parts to find $\int x \sin x \, dx$.

(b) A window is in the shape of a rectangle with a semicircle on top. The radius of the semicircle is r metres and the height of the rectangular part is x metres. The perimeter of the window is 20 metres.



(i) Use the perimeter to express x in terms of r and π .

(ii) Find, in terms of π , the value of r for which the area of the window is a maximum.

(c) The Maclaurin series for $\tan^{-1} x$ is $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

(i) Write down the general term of the series.

(ii) Use the Ratio Test to show that the series converges for $|x| < 1$.

(iii) Using the fact that $\frac{\pi}{4} = 4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$, and taking the first three terms in the Maclaurin series for $\tan^{-1} x$, find an approximation for π .
Give your answer correct to five decimal places.

ANSWERS

8 (a) $-x \cos x + \sin x + c$

(b) (i) $\frac{20 - 2r - \pi r}{2}$ (ii) $\frac{20}{\pi + 4}$

(c) (i) $(-1)^{n-1} \frac{x^{2n-1}}{2n-1}$ (iii) 3.17509