

CALCULUS OPTION (Q 8, PAPER 2)

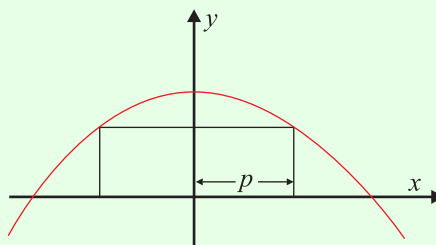
2010

8 (a) Use integration by parts to find $\int \log_e x \, dx$.

(b) A rectangle is inscribed between the curve $y = 9 - x^2$ and the x -axis, as shown.

(i) Write an expression for the area of the rectangle in terms of p .

(ii) Hence, calculate the area of the largest possible rectangle.



(c) (i) Derive the Maclaurin series for $f(x) = \cos x$ up to and including the term containing x^6 .

(ii) Hence, and using the identity $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, show that the first three

non-zero terms of the Maclaurin series for $\sin^2 x$ are $x^2 - \frac{x^4}{3} + \frac{2x^6}{45}$.

(iii) Use these terms to find an approximation for $\sin^2(\frac{1}{2})$, as a fraction.

ANSWERS

8 (a) $x \log_e x - x + c$

(b) (i) $18p - 2p^3$ (ii) $12\sqrt{3}$

(c) (i) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ (iii) $\frac{331}{1440}$