

**CALCULUS OPTION (Q 8, PAPER 2)**

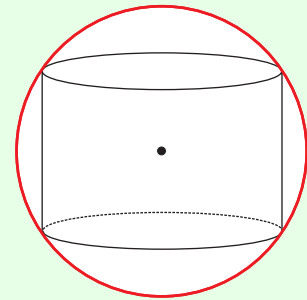
**2009**

8 (a) Use integration by parts to find  $\int xe^{4x} dx$ .

(b) (i) Derive the first four terms of the Maclaurin series for  $f(x) = \sqrt{1+x}$ .

(ii) Given that this series converges for  $-1 < x < 1$ , use these four terms to find an approximation for  $\sqrt{17}$ , as a fraction.

(c) The diagram shows a cylinder inscribed in a sphere. The cylinder has height  $2x$  and radius  $r$ . The sphere has fixed radius  $a$ .



(i) Express  $r$  in terms of  $a$  and  $x$ .

(ii) Find, in terms of  $a$ , the maximum possible volume of the cylinder.

**ANSWERS**

8 (a)  $\frac{1}{16} e^{4x} (4x - 1) + c$

(b) (i)  $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$  (ii)  $\frac{67553}{16384}$

(c) (i)  $r = \sqrt{a^2 - x^2}$  (ii)  $V_{\text{Max}} = \frac{4}{9} \sqrt{3} \pi a^3$