Calculus Option (Q 8, Paper 2)

2007

8 (a) p and q are real numbers such that $p + q = 1$.
Find the value of p that maximizes the product pq.

(b) (i) Derive the Maclaurin series for $f(x) = (1+x)^m$ up to an including the term containing x^3.

(ii) Given that the general term of the series $f(x)$ is

$$\frac{m(m-1)(m-2).......(m-r+1)}{r!}x^r,$$

show that the series converges for $-1 < x < 1$.

(c) Evaluate $\int_0^1 \tan^{-1} x\, dx$.

Answers

8 (a) $p = \frac{1}{2}$

(b) (i) $(1+x)^m = \binom{m}{0} + \binom{m}{1}x + \binom{m}{2}x^2 + \binom{m}{3}x^3$

(c) $\frac{\pi}{4} - \frac{1}{2}\ln 2$