CALCULUS OPTION (Q 8, PAPER 2)

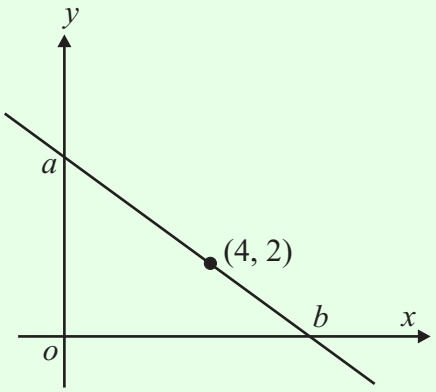
2006

8 (a) Derive the Maclaurin series for $f(x) = e^x$ up to and including the term containing x^3.

8 (b) A line passes through the point (4, 2) and has slope m, where $m < 0$. The line intersects the axes at the points a and b.

(i) Find the co-ordinates of a and b, in terms of m.

(ii) Hence, find the value of m for which the area of triangle aob is a minimum.



8 (c) Use the ratio test to test each of the following series for convergence. In each case, specify clearly the range of values of x for which the series converges, the range of values for which it diverges, and the value(s) of x for which the test is inconclusive.

(i) $\sum_{n=1}^{\infty} n 3^n x^n$ (ii) $\sum_{n=1}^{\infty} \frac{(n+1)!n!}{(2n)!} x^n$.

ANSWERS

8 (a) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

8 (b) (i) $a(0,\ 2-4m)$, $b(4-\frac{2}{m}, 0)$ (ii) $m = -\frac{1}{2}$

8 (c) (i) Convergent: $|3x| < 1 \Rightarrow -\frac{1}{3} < x < \frac{1}{3}$

Divergent: $|3x| > 1 \Rightarrow x > \frac{1}{3}, x < -\frac{1}{3}$

Inconclusive: $|3x| = 1 \Rightarrow x \pm \frac{1}{3}$

(ii) Convergent: $|\frac{x}{4}| < 1 \Rightarrow -4 < x < 4$

Divergent: $|\frac{x}{4}| > 1 \Rightarrow x > 4, x < -4$

Inconclusive: $|\frac{x}{4}| = 1 \Rightarrow x = \pm 4$