CALCULUS OPTION (Q 8, PAPER 2)

2004

8 (a) Use integration by parts to find $\int x \sin x \, dx$.

8 (b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + ..$ is the Maclaurin series.

(i) Derive the first five terms of the Maclaurin series for e^x.

(ii) Hence write down the first five terms of the Maclaurin series for e^{-x} and deduce the first three non-zero terms of the series for $\frac{e^x + e^{-x}}{2}$.

(iii) Write the general term of the series for $\frac{e^x + e^{-x}}{2}$ and use the Ratio Test to show that the series converges for all x.

8 (c) A solid cylinder has height h and radius r. The height of the cylinder, added to the circumference of its base, is equal to 3 metres.

(i) Express the volume of the cylinder in terms of r and π.

(ii) Find the maximum possible volume of the cylinder in terms of π.

ANSWERS

8 (a) $-x\cos x + \sin x + c$

8 (b) (i) $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ (ii) $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$ (iii) $u_n = \frac{x^{2n-2}}{(2n-2)!}$

8 (c) (i) $V = 3\pi r^2 - 2\pi^2 r^3$ (ii) $\frac{1}{\pi}$