

CALCULUS OPTION (Q 8, PAPER 2)

2003

8 (a) Use integration by parts to find $\int xe^{-5x} dx$.

8 (b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$ is the Maclaurin series.

(i) Derive the Maclaurin series for $f(x) = \log_e(1+x)$ up to an including the term containing x^4 .

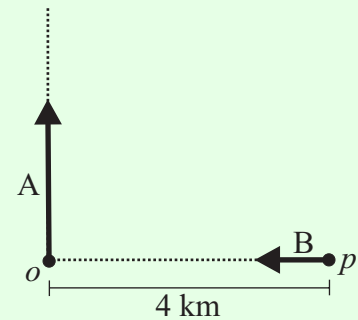
(ii) Write down the general term and use the Ratio Test to show that the series converges for $-1 < x < 1$.

8 (c) The point p is 4 km due east of the point o . At noon, A leaves o and travels north at a steady speed of 12 km/h. At the same time, B leaves p and travels towards o at a steady speed of 6 km/h.

(i) Write down expressions in x for the distances that A and B will each have travelled at x minutes after noon.

(ii) Find an expression in x for the distance that B will be from A at x minutes after noon.

(iii) At how many minutes after noon will B be closest to A?



ANSWERS

8 (a) $-\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x} + c$

8 (b) (i) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$ (ii) $u_n = (-1)^{n-1} \frac{x^n}{n}$

8 (c) (i) A: $\frac{x}{5}$ km; B: $\frac{x}{10}$ km

(ii) $\sqrt{(\frac{x}{5})^2 + (4 - \frac{x}{10})^2}$

(iii) 8 minutes