

CALCULUS OPTION (Q 8, PAPER 2)

2001

8 (a) Use integration by parts to find $\int x \cos x \, dx$.

8 (b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$ is the Maclaurin series for $f(x)$.

(i) Derive the Maclaurin series for $f(x) = \sin x$ up to and including the term containing x^7 .

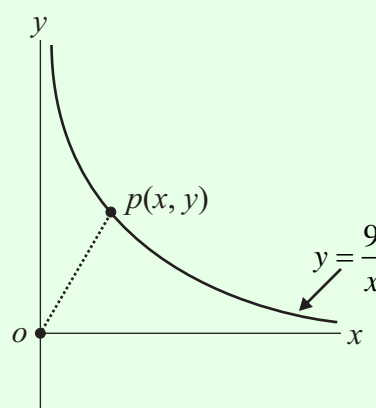
(ii) Write down the general term and use the Ratio Test to show that the series converges for all $x \in \mathbf{R}$.

8 (c) o is the origin, $(0, 0)$. $p(x, y)$ is a point on the curve $y = \frac{9}{x}$, where $x > 0$. $|op|$ is the distance from the origin to p .

(i) Express $|op|$ in terms of x .

(ii) Given that there is one value of x for which $|op|$ is a minimum, find this value of x .

(iii) Hence, find the minimum value of $|op|$.

**ANSWERS**

8 (a) $x \sin x + \cos x + c$

8 (b) (i) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ (ii) $u_n = (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}$

8 (c) (i) $|op| = \sqrt{x^2 + \frac{81}{x^2}}$ (ii) $x = 3$ (iii) $3\sqrt{2}$