

CALCULUS OPTION (Q 8, PAPER 2)

1999

8 (a) Use integration by parts to find $\int xe^x dx$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$ is the Maclaurin series.

Derive the Maclaurin series for $f(x) = \cos x$ up to and including the term containing x^6 .

Hence write down the general term of the Maclaurin series for $f(x) = \cos x$.

Use the ratio test to show that the series converges for all $x \in \mathbf{R}$.

(c) A solid cylinder of radius r and height h has a fixed volume K .

(i) Express h in terms of r , π and K .

(ii) Find the ratio $r:h$ when the total surface area of the cylinder is a minimum.
Give your answer as a ratio of natural numbers.

ANSWERS

8 (a) $xe^x - e^x + c$

8 (b) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}; u_n = \frac{(-1)^{n-1} x^{2n-2}}{(2n-2)!}$

8 (c) (i) $\frac{K}{\pi r^2}$ (ii) 1:2