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CALCULUS OPTION (Q 8, PAPER 2)

1997

8 (a) Use the ratio test to show that

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$

is convergent for all $x \in \mathbf{R}$.

(b) $f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \ldots$ is the Maclaurin series.

Write the first four terms of the Maclaurin series for

$f(x) = \sqrt{1+x}$.

As your expansion converges for $-1 < x < 1$, use it to evaluate $\sqrt{10}$ correct to one place of decimals.

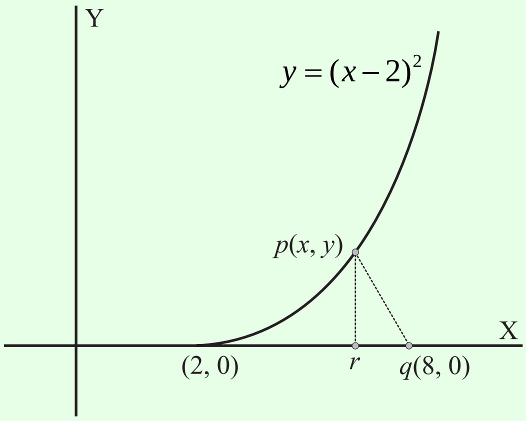
(c) $p(x, y)$ is a point on the curve $y = (x-2)^2$ in the domain $2 < x < 8$.

q is the point (8, 0) and $pr \perp rq$.

Express in terms of x, the area of the triangle prq.

What value of x maximises the area of triangle prq?

Find the maximum area of triangle prq.



ANSWERS

8 (b) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$, $3 \cdot 2$

(c) $\frac{1}{2}(8-x)(x-2)^2$, 6, 16