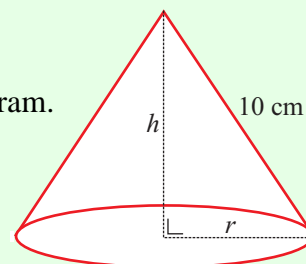


**CALCULUS OPTION (Q 8, PAPER 2)**

**1996**

8 (a) Use integration by parts to find  $\int xe^{-x} dx$ .

(b) The slant length of a right circular cone is 10 cm, see diagram.  
Find the maximum volume of the cone, in terms of  $\pi$ .



(c) Find  $f(0)$ ,  $f'(0)$ ,  $f''(0)$ ,  $f'''(0)$  for

$$f(x) = (1+x)^m.$$

Hence write the first four terms and the  $(r+1)$ th term of the Maclaurin series for

$$f(x) = (1+x)^m.$$

Test the series for convergence when  $m \in \mathbb{Q} \setminus \mathbb{N}$ .

**ANSWERS**

8 (a)  $-xe^{-x} - e^{-x} + c$

(b)  $\frac{2000\pi}{9\sqrt{3}} \text{ cm}^3$

(c)  $f(0) = 1$ ,  $f'(0) = m$ ,  $f''(0) = m(m-1)$ ,  $f'''(0) = m(m-1)(m-2)$

$$f(x) = 1 + mx + \frac{m(m-1)}{2} x^2 + \frac{m(m-1)(m-2)}{3} x^3$$

$$u_{r+1} = \frac{m(m-1)(m-2)\dots(m-r+1)}{r!} x^r; \text{ Convergent } |x| < 1$$