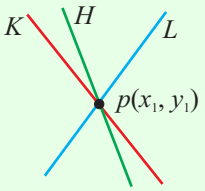
LINE (Q 3, PAPER 2)

LESSON NO. 5: CONCURRENT LINES

2005

3 (a) The line L_1: $5x + y + 3 = 0$ and the line L_2: $5x + y + 3 = 0$ intersect at the point p. Find the equation of the line through p perpendicular to L_2.

SOLUTION



CONCURRENT LINE FORMULA: Given any two concurrent lines, K and L, any other line, H, concurrent with these can be expressed as: $H = \mu K + \lambda L$ where $\mu,\ \lambda \in \mathbf{R}$.

Equation of new line: $\mu L_1 + \lambda L_2 = 0 \Rightarrow \mu(3x - 2y + 7) + \lambda(5x + y + 3) = 0$

$$\Rightarrow (3\mu + 5\lambda)x + (-2\mu + \lambda)y + 7\mu + 3\lambda = 0 \Rightarrow m = \frac{3\mu + 5\lambda}{2\mu - \lambda}$$

This line is perpendicular to L_2:

$$\Rightarrow \frac{3\mu + 5\lambda}{2\mu - \lambda} = \frac{1}{5} \Rightarrow 15\mu + 25\lambda = 2\mu - \lambda \Rightarrow 13\mu = -26\lambda \Rightarrow \mu = -2\lambda$$

Substitute this value of μ into the equation of the new line:

$$\Rightarrow (-6\lambda + 5\lambda)x + (4\lambda + \lambda)y - 14\lambda + 3\lambda = 0 \Rightarrow -\lambda x + 5\lambda y - 11\lambda = 0$$

$$\Rightarrow x - 5y + 11 = 0$$