LINE (Q 3, PAPER 2)

LESSON NO. 4: PERPENDICULAR DISTANCE

2004

3 (b) (i) Calculate the perpendicular distance from the point $(-1, -5)$ to the line $3x-4y-2=0$.

(ii) The point $(-1, -5)$ is equidistant from the lines $3x-4y-2=0$ and $3x-4y+k=0$, where $k \neq -2$. Find the value of k.

SOLUTION

3 (b) (i)

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$ 8

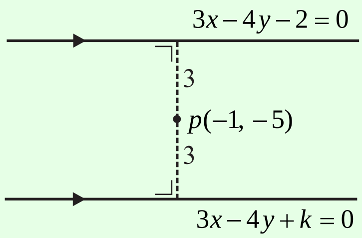
$$d=\frac{|3(-1)-4(-5)-2|}{\sqrt{3^2+(-4)^2}}=\frac{|-3+20-2|}{\sqrt{9+16}}=\frac{|15|}{\sqrt{25}}=3$$

3 (b) (ii)

$$3=\frac{|3(-1)-4(-5)+k|}{\sqrt{9+16}} \Rightarrow 3=\frac{|k+17|}{5} \Rightarrow 15=|k+17|$$

$$\Rightarrow \pm 15=k+17 \Rightarrow k=-32, -2$$

Ans: $k=-32$



2003

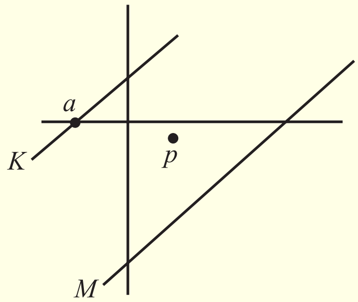
3 (b) K is the line $3x-4y+9=0$. The point $a(-3, 0)$ is on K.

The line M is parallel to K. The point $p(2, -1)$ is midway between K and M.

(i) Find the equation of M.

(ii) Calculate the distance between K and M.

(iii) Calculate the measure of the acute angle between ap and K. Give your answer correct to the nearest degree.



(iv) $b(x, y)$ is a point on K such that $|ab|=15$ and $x>0$. Find the value of x and the value of y.

SOLUTION

3 (b) (i)

Find a point on M by translating the point a through p.

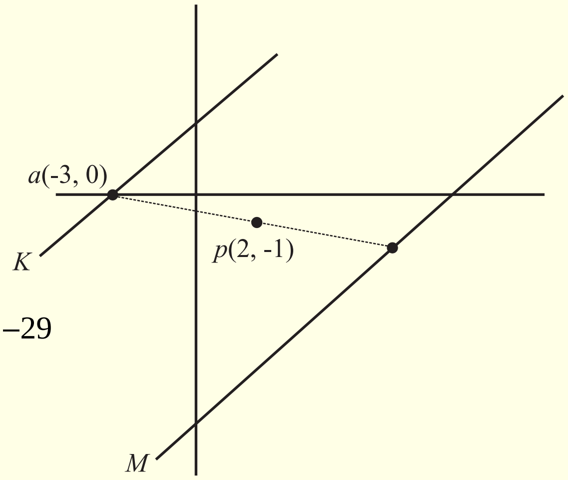
$a(-3, 0) \to p(2, -1) \to (7, -2)$

M has the same slope as K.

Equation of M: $m=\frac{3}{4}$, $(7, -2)$

$\therefore 3x-4y+k=0 \Rightarrow 3(7)-4(-2)+k=0 \Rightarrow k=-29$

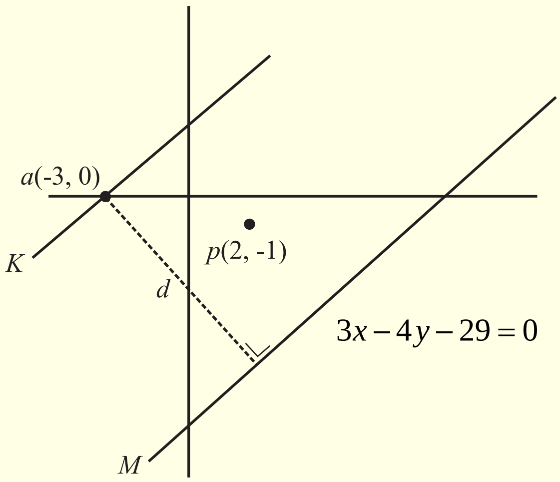
M: $3x-4y-29=0$



3 (b) (ii)

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$ 8

$$d=\frac{|3(-3)-4(0)-29|}{\sqrt{9+16}}=\frac{|-9+0-29|}{5}=\frac{38}{5}$$



3 (b) (iii)

$$\tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) \quad \ldots\ldots\ 7$$

Slope of ap: $m_1 = \dfrac{0+1}{-3-2} = -\dfrac{1}{5}$

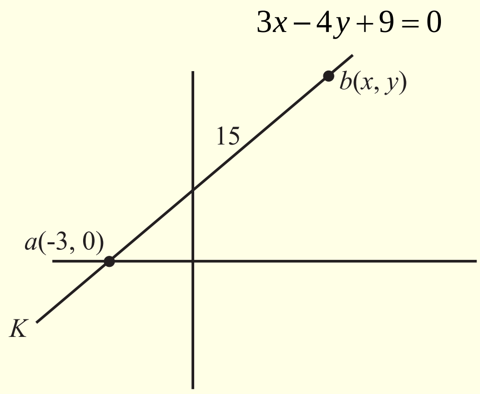
Slope of K: $m_2 = \dfrac{3}{4}$

$$\tan\theta = \left|\frac{-\frac{1}{5} - \frac{3}{4}}{1 + (-\frac{1}{5})(\frac{3}{4})}\right| \Rightarrow \tan\theta = \left|\frac{-4-15}{20-3}\right| = \left|\frac{-19}{17}\right|$$

$$\Rightarrow \theta = \tan^{-1}(\tfrac{19}{17}) = 48^\circ$$

3 (b) (iv)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots\ 1$$



$$15 = \sqrt{(x+3)^2 + (y-0)^2} \Rightarrow 225 = (x+3)^2 + y^2$$

$$b \in K \Rightarrow y = \frac{3x+9}{4}$$

$$\therefore 225 = (x+3)^2 + \left(\frac{3x+9}{4}\right)^2$$

$$\Rightarrow 225 = x^2 + 6x + 9 + \left(\frac{9x^2 + 54x + 81}{16}\right)$$

$$\Rightarrow 3600 = 16x^2 + 96x + 144 + 9x^2 + 54x + 81$$

$$\Rightarrow 0 = 25x^2 + 150x - 3375 \Rightarrow x^2 + 6x - 135 = 0$$

$$\Rightarrow (x-9)(x+15) = 0 \Rightarrow x = -15,\ 9$$

As $x > 0 \Rightarrow x = 9,\ y = \dfrac{3(9)+9}{4} = 9$

Ans: (9, 9)