LINE (Q 3, PAPER 2)

LESSON NO. 3: ANGLE BETWEEN LINES

2006

3 (c) (i) Prove that the measure of one of the angles between two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

SOLUTION

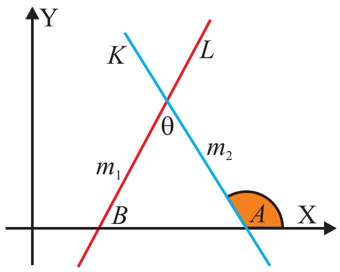
PROOF

$A = B + \theta \Rightarrow \theta = (A - B)$

$\therefore \tan\theta = \tan(A - B)$

$\Rightarrow \tan\theta = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\Rightarrow \tan\theta = \pm\left(\dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$



$\pm$ is to take account of acute and obtuse cases since $\tan(180 - \theta) = -\tan\theta$.

2001

3 (c) N is the line $tx+(t-2)y+4=0$ where $t \in \mathbf{R}$.

(i) Write down the slope of N in terms of t.

(ii) Given that the angle between N and the line $x-3y+1=0$ is 45°, find the two possible values of t.

SOLUTION

3 (c)

N: $tx+(t-2)y+4=0$

3 (c) (i)

If you are given the equation of a line you can write down its slope:

$$ax+by+c=0 \Rightarrow m=-\tfrac{a}{b}$$

Slope of N: $m_1 = -\dfrac{t}{t-2} = \dfrac{t}{2-t}$

3 (c) (ii)

$$\tan\theta = \pm\left(\frac{m_1-m_2}{1+m_1m_2}\right) \text{ .... } 7$$

L: $x-3y+1=0 \Rightarrow m_2 = \frac{1}{3}$

$$\tan 45^{\circ} = \left|\frac{\frac{t}{2-t}-\frac{1}{3}}{1+(\frac{t}{2-t})(\frac{1}{3})}\right| \Rightarrow 1 = \left|\frac{\frac{t}{2-t}-\frac{1}{3}}{1+(\frac{t}{2-t})(\frac{1}{3})}\right| \times \frac{3(2-t)}{3(2-t)}$$

$$\Rightarrow 1 = \left|\frac{3t-(2-t)}{3(2-t)+t}\right| \Rightarrow 1 = \left|\frac{4t-2}{6-2t}\right| \Rightarrow 1 = \left|\frac{2t-1}{3-t}\right|$$

$$\Rightarrow \pm 1 = \frac{2t-1}{3-t} \Rightarrow \pm 1(3-t) = (2t-1)$$

$$3-t = 2t-1 \Rightarrow 4 = 3t \Rightarrow t = \tfrac{4}{3} \text{ or } -3+t = 2t-1 \Rightarrow t = -2$$

Ans: $t=-2, \frac{4}{3}$