LINE (Q 3, PAPER 2)

2008

3 (a) The parametric equations $x = 7t - 4$ and $y = 3 - 3t$ represents a line, where $t \in \mathbf{R}$. Find the Cartesian equation of the line.

(b) $a(2, 1)$, $b(10, 7)$, $c(14, 10)$ and $d(7, 1)$ are four points.

(i) Plot a, b, c and d on the co-ordinate plane.

(ii) Verify that $|ab| = 2|bc|$ and $|ab| = 2|ad|$.

(iii) Find a', b', c' and d', the repective images of a, b, c and d under the transformation $f:(x, y) \to (x', y')$, where $x' = x + y$ and $y' = x - 2y$.

(iv) Verify that $|a'b'| = 2|b'c'|$ but that $|a'b'| \neq 2|a'd'|$.

(c) Prove that the perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

SOLUTION

3 (a)

Eliminate t from each equation and then equate both equations.

$x = 7t - 4 \Rightarrow t = \dfrac{x+4}{7}$

$y = 3 - 3t \Rightarrow t = \dfrac{3-y}{3}$

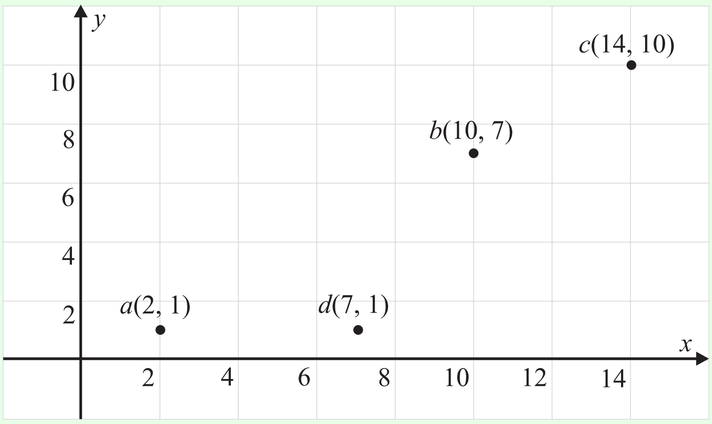
$\therefore \dfrac{x+4}{7} = \dfrac{3-y}{3}$ [Multiply across by 21.]

$\Rightarrow 3(x+4) = 7(3-y)$

$\Rightarrow 3x + 12 = 21 - 7y$

$\therefore 3x + 7y - 9 = 0$

3 (b) (i)



3 (b) (ii)

$|ab| = \sqrt{(10-2)^2+(7-1)^2} = \sqrt{64+36} = \sqrt{100} = 10$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ **1**

$|bc| = \sqrt{(14-10)^2+(10-7)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\therefore |ab| = 2|bc|$

$|ad| = \sqrt{(7-2)^2+(1-1)^2} = \sqrt{25+0} = \sqrt{25} = 5$

$\therefore |ab| = 2|ad|$

3 (b) (iii)

$x' = x+y,\ y' = x-2y$

$a(2, 1) \to a'(2+1,\ 2-2(1)) = a'(3, 0)$

$b(10, 7) \to b'(10+7,\ 10-2(7)) = b'(17, -4)$

$c(14, 10) \to c'(14+10,\ 14-2(10)) = c'(24, -6)$

$d(7, 1) \to d'(7+1,\ 7-2(1)) = d'(8, 5)$

3 (b) (iv)

$|a'b'| = \sqrt{(17-3)^2+(-4-0)^2} = \sqrt{196+16} = \sqrt{212} = 2\sqrt{53}$

$|b'c'| = \sqrt{(24-17)^2+(-6-(-4))^2} = \sqrt{49+4} = \sqrt{53}$

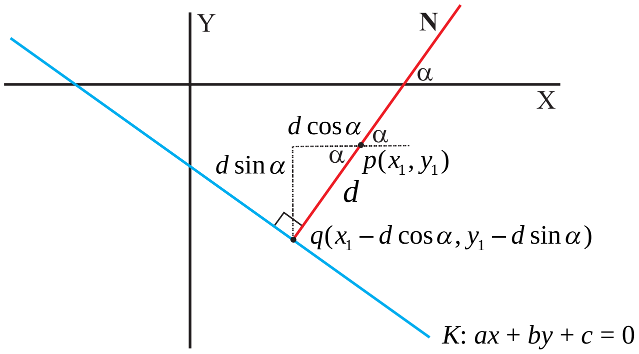
$\therefore |a'b'| = 2|b'c'|$

$|a'd'| = \sqrt{(8-3)^2+(5-0)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$\therefore |a'b'| \neq 2|a'd'|$

3 (c)

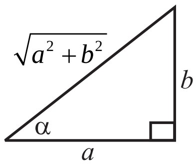
PROOF: PERPENDICULAR DISTANCE FORMULA



1. $q \in K \Rightarrow \; a(x_1 - d\cos\alpha) + b(y_1 - d\sin\alpha) + c = 0$

$\Rightarrow ax_1 + by_1 + c = d(a\cos\alpha + b\sin\alpha)$

2. Slope of **N** = $\tan\alpha = \frac{b}{a}$ since **N** $\perp$ K



$$\therefore a\cos\alpha + b\sin\alpha = \frac{a \times a}{\sqrt{a^2 + b^2}} + \frac{b \times b}{\sqrt{a^2 + b^2}}$$

$$= \frac{a^2 + b^2}{\sqrt{a^2 + b^2}} = \sqrt{a^2 + b^2}$$

3. $$\therefore d = \frac{|ax_1 + by_1 + c|}{a\cos\alpha + b\sin\alpha} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$