LINE (Q 3, PAPER 2)

2000

3 (a) The equation of the line L is $14x + 6y + 1 = 0$.
Find the equation of the line perpendicular to L that contains the point $(3, -2)$.

3 (b) $a(1, -2)$ and $c(-4, 8)$ are two points.
f is the transformation $(x, y) \to (x', y')$ where $x' = 2x - 3y$ and $y' = 6x + y$.
(i) b divides $[ac]$ in the ratio 3:2. Find the coordinates of b.

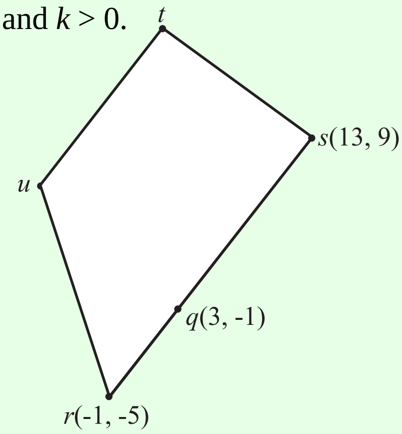
(ii) Find $f(a)$, $f(b)$ and $f(c)$.

(iii) Verify that $|f(a)f(b)| : |f(b)f(c)| = |ab| : |bc|$.

3 (c) $rstu$ is a quadrilateral where r is $(-1, -5)$ and s is $(13, 9)$.
$q(3, -1)$ lies between r and s.

(i) The coordinates of u are $(-2k, 3k)$ where $k \in \mathbf{R}$ and $k > 0$.
The area of the triangle rqu is 28 square units.
Find the value of k.

(ii) The slope of ts is $-\frac{3}{11}$.
sr is parallel to tu.
Find the coordinates of t.



SOLUTION

3 (a)

$L: 14x + 6y + 1 = 0 \Rightarrow m = -\frac{14}{6} = -\frac{7}{3}$

Slope of perpendicular line: $m = \frac{3}{7}$

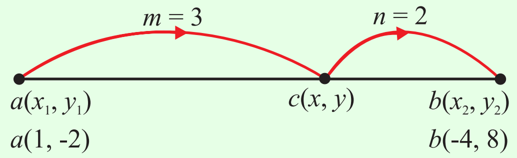
> 2. **PERPENDICULAR LINES**
> Two lines are perpendicular if the product of their slopes is -1.
> $K \perp L \Leftrightarrow m_1 \times m_2 = -1$. If $m_1 = -\frac{a}{b} \Rightarrow m_2 = \frac{b}{a}$

Equation of perpendicular line: Point $(3, -2)$, $m = \frac{3}{7}$

$3x - 7y + k = 0 \Rightarrow 3(3) - 7(-2) + k = 0 \Rightarrow k = -23$

$\therefore 3x - 7y - 23 = 0$

3 (b) (i)



$$x=\frac{mx_2+nx_1}{m+n},\ y=\frac{my_2+ny_1}{m+n}$$ **5**

$$x=\frac{3(-4)+2(1)}{5}=-2,\ y=\frac{3(8)+2(-2)}{5}=4$$

Ans: $b(-2,\ 4)$

3 (b) (ii)

$$a(1,\ -2): x'=2(1)-3(-2)=8;\ y'=6(1)+(-2)=4 \Rightarrow f(a)=(8,\ 4)$$

$$b(-2,\ 4): x'=2(-2)-3(4)=-16;\ y'=6(-2)+(4)=-8 \Rightarrow f(b)=(-16,\ -8)$$

$$c(-4,\ 8): x'=2(-4)-3(8)=-32;\ y'=6(-4)+(8)=-16 \Rightarrow f(c)=(-32,\ -16)$$

3 (b) (iii)

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$ **1**

$$|f(a)f(b)|=\sqrt{(-8-4)^2+(-16-8)^2}=12\sqrt{5}$$

$$|f(b)f(c)|=\sqrt{(-32+16)^2+(-16+8)^2}=8\sqrt{5}$$

$$|ab|=\sqrt{(-2-1)^2+(4+2)^2}=3\sqrt{5}$$

$$|bc|=\sqrt{(-4+2)^2+(8-4)^2}=2\sqrt{5}$$

$$\therefore |f(a)f(b)|:|f(b)f(c)|=12\sqrt{5}:8\sqrt{5}=3:2$$

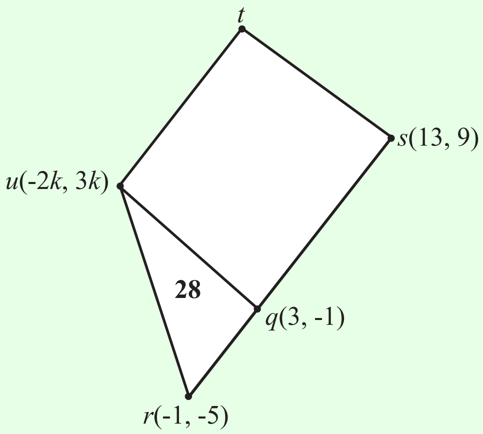
$$\therefore |ab|:|bc|=3\sqrt{5}:2\sqrt{5}=3:2$$

$$\therefore |f(a)f(b)|:|f(b)f(c)|=|ab|:|bc|$$

3 (c) (i)

$$A=\tfrac{1}{2}|x_1y_2-x_2y_1|$$ **4**

Steps
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.



Area of triangle *rqu* is 28 square units.

$$r(-1,\ -5)\to(-4,\ -4)$$

$$q(3,\ -1)\to(0,\ 0)$$

$$u(-2k,\ 3k)\to(-2k-3,\ 3k+1)$$

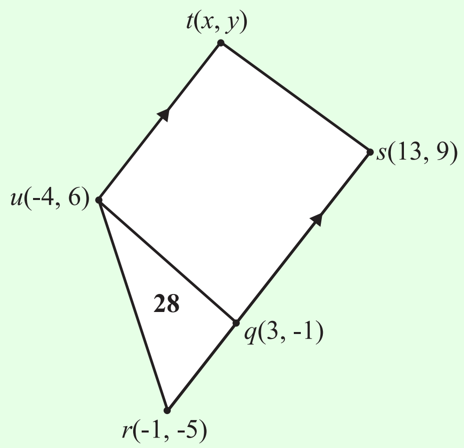
$$\Rightarrow A=\tfrac{1}{2}|-4(3k+1)-(-4)(-2k-3)|=28$$

$$\Rightarrow |-6k-2-4k-6|=28$$

$$\Rightarrow |-10k-8|=28 \Rightarrow -10k-8=\pm 28$$

$$\therefore k=2$$

3 (c) (ii)



$m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ **2**

Slope of *sr*: $m = \dfrac{-5-9}{-1-13} = \dfrac{-14}{-14} = 1$

Slope of *tu*: m = 1 [Parallel]

Slope of *ts*: $m = -\frac{3}{11}$

Let the coordinates of *t* by (*x*, *y*).

Slope of *tu*: $\dfrac{y-6}{x+4} = 1 \Rightarrow y - 6 = x + 4 \Rightarrow x - y = -10 ...(\mathbf{1})$

Slope of *ts*: $\dfrac{9-y}{13-x} = -\dfrac{3}{11} \Rightarrow 11(9-y) = -3(13-x) \Rightarrow 3x + 11y = 138 ...(\mathbf{2})$

Solve equations (**1**) and (**2**) simultaneously.

$$\begin{aligned} x - y &= -10 ...(\mathbf{1})(\times -3) \\ 3x + 11y &= 138 ...(\mathbf{2}) \end{aligned}$$

$$\begin{aligned} -3x + 3y &= 30 \\ 3x + 11y &= 138 \\ \hline 14y &= 168 \Rightarrow y = 12 \end{aligned}$$

Substitute this value of *y* into Eqn. (**1**).

$x - 12 = -10 \Rightarrow x = 2$

Ans: *t*(2, 12)