## Line (Q 3, Paper 2)

### 1999

3 (a) Show that the line $6x - 8y - 71 = 0$ contains the midpoint $[ab]$ where $a$ has coordinates $(8, -6)$ and $b$ has coordinates $(5, -2)$.

(b) The line $L$ has equation $5x - 3y + 10 = 0$.
The point $k$ has coordinates (6, 2).
Show that the perpendicular distance from $k$ to $L$ is $\sqrt{34}$.
$f$ is the transformation $(x, y) \to (x', y')$ where

$$x' = 7x - 2y$$
$$y' = -4x + y.$$

The image of $L$ under $f$ is the line $f(L)$. Find the equation of $f(L)$.

Show that the perpendicular distance from $f(k)$ to $f(L)$ is $\frac{\sqrt{34}}{\sqrt{5}}$.

(c) A line containing the point $(-4, -2)$ has slope $m$, where $m \neq 0$.
This line intercepts the $x$ axis at $(x_1, 0)$ and the $y$ axis at $(0, y_1)$.
Given that $x_1 + y_1 = 3$, find the slopes of the two lines that satisfy this condition.
Find the measure of the acute angle between these two lines and give your answer to the nearest degree.

**Solution**

**3 (a)**

Midpoint of $[ab] = \left(\frac{8+5}{2}, \frac{-6-2}{2}\right) = (\frac{13}{2}, -4)$

Is $(\frac{13}{2}, -4) \in L$?

$6(\frac{13}{2}) - 8(-4) - 71$

$= 39 + 32 - 71$

$= 0 \Rightarrow (\frac{13}{2}, -4) \in L$

**3 (b)**

Point $k(6, 2)$, $L: 5x - 3y + 10 = 0$

$$d = \frac{5(6) - 3(2) + 10}{\sqrt{5^2 + (-3)^2}} = \frac{34}{\sqrt{34}} = \sqrt{34}$$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ ....... 8

$$\begin{array}{ll} x' = 7x - 2y & \\ 2y' = -8x + 2y & \\ \hline -x' - 2y' = x & \end{array} \qquad \Leftarrow x' = 7x - 2y \Rightarrow \quad \Leftarrow y' = -4x + y \Rightarrow \qquad \begin{array}{l} 4x' = 28x - 8y \\ 7y' = -28x + 7y \\ \hline -4x' - 7y' = y \end{array}$$

$L: 5x - 3y + 10 = 0$

$\Rightarrow f(L) = 5(-x' - 2y') - 3(-4x' - 7y') + 10 = 0$

$\Rightarrow -5x' - 10y' + 12x' + 21y' + 10 = 0$

$\therefore 7x' + 11y' + 10 = 0$

$k(6,\ 2) \to f(k) = (38,\ -22):\ x' = 7(6) - 2(2) = 38;\ y' = -4(6) + (2) = -22$

Point $f(k) = (38,\ -22),\ f(L) = 7x' + 11y' + 10 = 0$

$$d = \frac{|7(38) + 11(-22) + 10|}{\sqrt{7^2 + 11^2}} = \frac{34}{\sqrt{170}} = \frac{34}{10\sqrt{17}} = \frac{\sqrt{34}}{\sqrt{5}}$$

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ ……. 8

**3 (c)**

Point $(-4,\ -2)$, slope $= +\frac{m}{1}$

Equation of the line $L$: $mx - y + k = 0$

$(-4,\ -2) \in L \Rightarrow -4m - (-2) + k = 0 \Rightarrow k = 4m - 2$

Therefore, equation of line $L$: $mx - y + 4m - 2 = 0$

$(x_1,\ 0) \in L \Rightarrow m(x_1) - (0) + 4m - 2 = 0 \Rightarrow mx_1 = -4m + 2$

$\therefore x_1 = \dfrac{-4m + 2}{m}$

$(0,\ y_1) \in L \Rightarrow m(0) - (y_1) + 4m - 2 = 0$

$\therefore y_1 = 4m - 2$

$x_1 + y_1 = 3 \Rightarrow \left(\dfrac{-4m + 2}{m}\right) + 4m - 2 = 3$

$\Rightarrow -4m + 2 + 4m^2 - 2m - 3m = 0$

$\Rightarrow 4m^2 - 9m + 2 = 0$

$\Rightarrow (4m - 1)(m - 2) = 0$

$\therefore m = \frac{1}{4},\ 2$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{(\frac{1}{4} - 2)}{(1 + (\frac{1}{4})(2))} \times \frac{4}{4}\right| = \left|\frac{1 - 8}{4 + 2}\right| = \frac{7}{6}$$

$\tan\theta = \pm\left(\dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$ ……. 7

$\therefore \theta = \tan^{-1}(\frac{7}{6}) = 49^{\circ}$