LINE (Q 3, PAPER 2)

1998

3 (a) The parametric equations $x = 3 - 4t$ and $y = 1 + 2t$ represent a line, where $t \in \mathbf{R}$. Find the Cartesian equation of the line.

(b) Find the equation of the line pq where p has coordinates $(7, -6)$ and q has coordinates $(-3, 2)$.

Find the point of intersection of pq and the line $2x - 3y + 1 = 0$.

Determine the ratio in which the line $2x - 3y + 1 = 0$ divides $[pq]$.

(c) (i) The line M is $ax + by + c = 0$.

Prove that the perpendicular distance from the point (x_1, y_1) to the line M is given by

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

(ii) If p is the length of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$,

prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

SOLUTION

3 (a)

Write each parametric equation in terms of t and then equate.

$x = 3 - 4t \Rightarrow t = \frac{3 - x}{4}$

$y = 1 + 2t \Rightarrow t = \frac{y - 1}{2}$

$\Rightarrow \frac{3 - x}{4} = \frac{y - 1}{2}$ [Multiply across by 4.]

$\Rightarrow 3 - x = 2(y - 1)$

$\Rightarrow 3 - x = 2y - 2$

$\therefore x + 2y - 5 = 0$

3 (b)

Equation of pq: Points $(7, -6), (-3, 2)$

$m = \frac{2 - (-6)}{-3 - 7} = \frac{8}{-10} = -\frac{4}{5}$ $\quad$ $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ 2

Equation of pq: $4x + 5y + k = 0$

$(-3, 2) \in pq \Rightarrow 4(-3) + 5(2) + k = 0 \Rightarrow k = 2$

$\therefore 4x + 5y + 2 = 0$

[F] INTERSECTING LINES

To find out where two lines intersect, solve their equations simultaneously.

$$4x+5y+2=0 \ldots(\mathbf{1})$$
$$2x-3y+1=0 \ldots(\mathbf{2})(\times -2)$$

$\rightarrow$

$$\begin{aligned} 4x+5y+2&=0 \\ -4x+6y-2&=0 \\ \hline 11y \quad &=0 \Rightarrow y=0 \end{aligned}$$

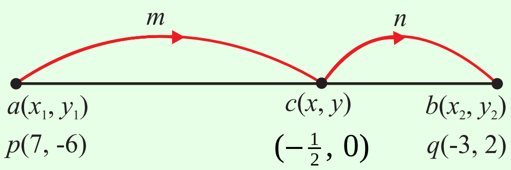
Substitute this value of y into Eqn. (**1**): $\therefore 4x+5(0)+2=0 \Rightarrow x=-\frac{1}{2}$

Point of intersection: $(-\frac{1}{2},\ 0)$

Using $y=\dfrac{my_2+ny_1}{m+n}$

$\therefore 0=\dfrac{2m-6n}{m+n} \Rightarrow 2m=6n \Rightarrow m=3n$

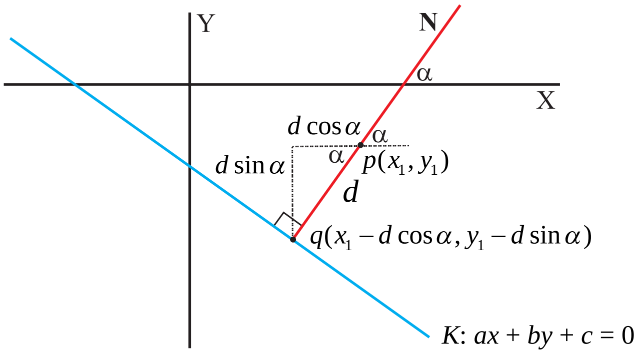
$\therefore \dfrac{m}{n}=\dfrac{3}{1} \Rightarrow m:n=3:1$



$$x=\frac{mx_2+nx_1}{m+n},\ y=\frac{my_2+ny_1}{m+n} \quad \ldots\ldots\ \mathbf{5}$$

3 (c) (i)

PROOF: PERPENDICULAR DISTANCE FORMULA



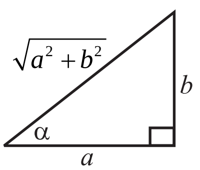
1. $q \in K \Rightarrow \ a(x_1-d\cos\alpha)+b(y_1-d\sin\alpha)+c=0$

$\Rightarrow ax_1+by_1+c=d(a\cos\alpha+b\sin\alpha)$

2. Slope of **N** = $\tan\alpha=\frac{b}{a}$ since **N** $\perp$ K

$$\therefore a\cos\alpha+b\sin\alpha=\frac{a\times a}{\sqrt{a^2+b^2}}+\frac{b\times b}{\sqrt{a^2+b^2}}$$

$$=\frac{a^2+b^2}{\sqrt{a^2+b^2}}=\sqrt{a^2+b^2}$$



3. $$\therefore d=\frac{|ax_1+by_1+c|}{a\cos\alpha+b\sin\alpha}=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

3 (c) (ii)

Point (0, 0)

Perpendicular distance = p

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \quad \ldots\ldots\ 8$$

Line: $\frac{x}{a} + \frac{y}{b} = 1$ [Multiply across by *ab*.]

$\Rightarrow bx + ay = ab$

$\therefore bx + ay - ab = 0$

$$\therefore p = \frac{|b(0) + a(0) - ab|}{\sqrt{b^2 + a^2}} \Rightarrow p = \frac{|-ab|}{\sqrt{b^2 + a^2}}$$

$$\Rightarrow p\sqrt{b^2 + a^2} = ab$$

$$\Rightarrow p^2(b^2 + a^2) = a^2b^2$$

$$\Rightarrow \frac{(b^2 + a^2)}{a^2b^2} = \frac{1}{p^2}$$

$$\therefore \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$$