1997

3 (a) A triangle has vertices $(1,-1), (5,1)$ and $(-\frac{5}{2},-5)$.
Find the area of the triangle.

(b) K_1 and K_2 are two lines with slopes m_1 and m_2, repectively.
If θ is an angle between K_1 and K_2, prove that

$$\tan\theta = \pm\left|\frac{m_1 - m_2}{1+m_1m_2}\right|.$$

(c) f is the transformation $(x, y) \to (x', y')$ where

$$x' = 4x - y$$
$$y' = 2x + y.$$

For the points $a(0,0), b(-2,-5)$ and $c(4, 9)$, find $f(a)$, $f(b)$ and $f(c)$.

(i) L is the line ac. The image of L under f is the line $f(L)$.
Find the equation of the $f(L)$.

(ii) $f(M)$ is the image of the line M under f.
$f(M)$ is perpendicular to $f(L)$ and $f(b) \in f(M)$.
Find the equation of the line M.

SOLUTION

3 (a)

$(1, -1) \to (0, 0)$

$(5, 1) \to (4, 2)$

$(-\frac{5}{2}, -5) \to (-\frac{7}{2}, -4)$

$A = \frac{1}{2}|x_1y_2 - x_2y_1|$ 4

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

$A = \frac{1}{2}|4(-4) - (2)(-\frac{7}{2})|$

$\Rightarrow A = \frac{1}{2}|-16+7| = \frac{1}{2}|-9|$

$\therefore A = \frac{9}{2}$ square units

3 (b)

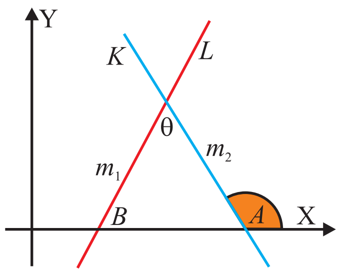
PROOF: ANGLE BETWEEN LINES FORMULA

$A = B + \theta \Rightarrow \theta = (A - B)$

$\therefore \tan\theta = \tan(A - B)$

$$\Rightarrow \tan\theta = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\Rightarrow \tan\theta = \pm\left(\frac{m_1 - m_2}{1 + m_1m_2}\right)$$



$\pm$ is to take account of acute and obtuse cases since $\tan(180 - \theta) = -\tan\theta$.

3 (c)

$x' = 4x - y;\ y' = 2x + y$

$a(0,\ 0) \to f(a) = (0,\ 0): x' = 4(0) - (0) = 0;\ y' = 2(0) + (0) = 0$

$b(-2,\ -5) \to f(b) = (-3,\ -9): x' = 4(-2) - (-5) = -3;\ y' = 2(-2) + (-5) = -9$

$c(4,\ 9) \to f(c) = (7,\ 17): x' = 4(4) - (9) = 7;\ y' = 2(4) + (9) = 17$

3 (c) (i)

Line L: ac

Line $f(L)$: $f(a)f(c)$

$f(a) = (0,\ 0),\ f(c) = (7,\ 17)$

$m = \tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ **2**

$$m = \frac{17-0}{7-0} = \frac{17}{7}$$

Eqn. of $f(L): 17x' - 7y' + k = 0$

$(0,\ 0) \in f(L) \Rightarrow 17(0) - 7(0) + k = 0 \Rightarrow k = 0$

$\therefore\ f(L): 17x' - 7y' = 0$

3 (c) (ii)

$f(M) \perp f(L)$

$\Rightarrow$ Slope of $f(M): m = -\frac{7}{17}$

Eqn. of $f(M): 7x' + 17y' + k = 0$

$f(b) = (-3,\ -9) \in f(M) \Rightarrow 7(-3) + 17(-9) + k = 0 \Rightarrow k = 174$

$\therefore\ f(M): 7x' + 17y' + 174 = 0$

$\therefore M: 7(4x - y) + 17(2x + y) + 174 = 0$

$\Rightarrow 28x - 7y + 34x + 17y + 174 = 0$

$\Rightarrow 62x + 10y + 174 = 0$

$\therefore 31x + 5y + 87 = 0$