1996

3 (a) (i) The parametric equations of the lines L and K are:

L: $x = t + \frac{1}{2}$, $y = 2t + 7$

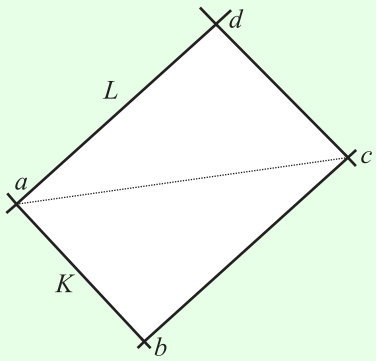
K: $x = \frac{1-t}{3}$, $y = t - 5$.

Show that their Cartesian equations are

L: $2x - y + 6 = 0$

K: $3x + y + 4 = 0$

and find a, their point of intersection.



(ii) If L and K contain adjacent sides of a parallelogram $abcd$ and the mid-point of $[ac]$ is $(0, 3\frac{1}{2})$, find the coordinates of vertices c, b and d.

(b) If $p = (0, 0)$ and $q = (1, 2)$ show that

$x = t,\ y = 2t,\ 0 \le t \le 1$

are parametric equations of the line segment $[pq]$.

Find the image of this segment under the transformation

$x' = 3x - y$

$y' = x - y$.

SOLUTION

3 (a) (i)

Eliminate the parameter t from one equation and substitute it into the other equation.

$L: x = t + \frac{1}{2} \Rightarrow t = x - \frac{1}{2}$

$y = 2t + 7 \Rightarrow y = 2(x - \frac{1}{2}) + 7$

$\Rightarrow y = 2x - 1 + 7$

$\Rightarrow y = 2x + 6$

$L: 2x - y + 6 = 0$

$K: y = t - 5 \Rightarrow t = y + 5$

$x = \frac{1-t}{3} \Rightarrow x = \frac{1-(y+5)}{3}$

$\Rightarrow 3x = 1 - y - 5$

$\Rightarrow 3x = -y - 4$

$K: 3x + y + 4 = 0$

Solve *K* and *L* simultaneously to find the point of intersection *a*.

$$\begin{aligned} 2x - y + 6 &= 0 \ldots(\mathbf{1}) \\ 3x + y + 4 &= 0 \ldots(\mathbf{2}) \\ \hline 5x \quad + 10 &= 0 \Rightarrow x = -2 \end{aligned}$$

Substitute this value of *x* into Eqn. (**1**): $2(-2) - y + 6 = 0 \Rightarrow -4 - y + 6 = 0$

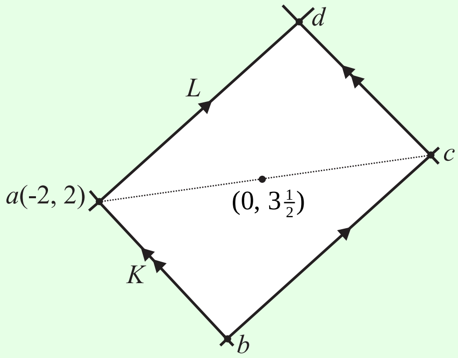
$\therefore y = 2$

$\therefore a(-2, 2)$ is the point of intersection.

3 (a) (ii)

You can find *c* by passing *a* by a central symmetry through the midpoint of [*ac*].

$a(-2, 2) \rightarrow (0, 3\tfrac{1}{2}) \rightarrow c(2, 5)$



To find *d*, find the equation of line *dc* and solve it simultaneously with *L*.

Equation of *dc*: Point *c*(2, 5), slope $m = -3$ $(cd \parallel K)$

$\Rightarrow 3x + y + k = 0$

$(2, 5) \in cd \Rightarrow 3(2) + (5) + k = 0 \Rightarrow k = -11$

$\therefore 3x + y - 11 = 0$

$$\begin{aligned} 2x - y + 6 &= 0 \ldots.(\mathbf{1}) \\ 3x + y - 11 &= 0 \ldots.(\mathbf{2}) \\ \hline 5x \quad - 5 &= 0 \Rightarrow x = -1 \end{aligned}$$

Substitute this value of *x* into Eqn. (**1**) to get *y*: $2(1) - y + 6 = 0 \Rightarrow -y = -8$

$\therefore y = 8 \Rightarrow d(1, 8)$

Pass *d* through the midpoint of [*ac*] by a central symmetry to get *b*.

$d(1, 8) \rightarrow (0, 3\tfrac{1}{2}) \rightarrow b(-1, -1)$

3 (b)

Parametric equation of a line segment:

$$\begin{aligned} x &= x_1 + t(x_2 - x_1) \\ y &= y_1 + t(y_2 - y_1) \end{aligned}$$

$p = (0, 0) = (x_1, y_1);\ q = (1, 2) = (x_2, y_2)$

$\therefore x = 0 + t(1 - 0) \Rightarrow x = t$

$\therefore y = 0 + t(2 - 0) \Rightarrow y = 2t$

$p = (0, 0) \rightarrow f(p) = (0, 0) : x' = 3(0) - (0) = 0;\ y' = 0 - 0 = 0$

$q = (1, 2) \rightarrow f(q) = (1, -1) : x' = 3(1) - (2) = 1;\ y' = 1 - 2 = -1$