LINE (Q 3, PAPER 2)

LESSON NO. 1: THE BASICS

2006

3 (a) Show that the line containing the points $(3, -6)$ and $(-7, 12)$ is perpendicular to the line $5x - 9y + 6 = 0$.

SOLUTION

3 (a)

Slope between two points: $m_1 = \frac{-6-12}{3+7} = -\frac{18}{10} = -\frac{9}{5}$

$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$ 2

Slope of the line: $m_2 = \frac{5}{9}$

$m_1 \times m_2 = \left(-\frac{9}{5}\right)\left(\frac{5}{9}\right) = -1$

Therefore, the two lines are perpendicular to each other.

2. **PERPENDICULAR LINES**
Two lines are perpendicular if the product of their slopes is -1.
$K \perp L \Leftrightarrow m_1 \times m_2 = -1$. If $m_1 = -\frac{a}{b} \Rightarrow m_2 = \frac{b}{a}$

2005

3 (a) The line L_1: $3x-2y+7=0$ and the line L_2: $5x+y+3=0$ intersect at the point p. Find the equation of the line through p perpendicular to L_2.

3 (b) The line K passes through the point $(-4, 6)$ and has slope m, where $m > 0$.

(i) Write down the equation of K in terms of m.

(ii) Find, in terms of m, the co-ordinates of the points where K intersects the axes.

(iii) The area of the triangle formed by K, the x-axis and the y-axis is 54 square units. Find the possible values of m.

SOLUTION

3 (a)

You can do this using two methods:

Method 1: Find the point of intersection p by solving L_1 and L_2 simultaneously.

$$\begin{aligned} 3x-2y&=-7 \ldots(\mathbf{1}) \\ 10x+2y&=-6 \ldots(\mathbf{2})(\times 2) \end{aligned} \quad \rightarrow \quad \begin{aligned} 3x-2y&=-7 \\ 10x+2y&=-6 \\ \hline 13x \quad &=-13 \Rightarrow x=-1 \end{aligned}$$

Substitute this value for x into equation ($\mathbf{1}$): $3(-1)-2y=-7 \Rightarrow -2y=-4 \Rightarrow y=2$

Therefore, point of intersection $p(-1, 2)$.

L_2: $5x+y+3=0 \Rightarrow m=-5 \Rightarrow m^{\perp}=\frac{1}{5}$

New line: $m=\frac{1}{5}$, $p(-1, 2)$

Equation of new line: $x-5y+k=0 \Rightarrow (-1)-5(2)+k=0 \Rightarrow -11+k=0 \Rightarrow k=11$

Ans: $x-5y+11=0$

3 (b) (i)

Slope $=+\frac{m}{1}$, Point $(-4, 6)$

Equation of K: $mx-y+k=0 \Rightarrow m(-4)-(6)+k=0 \Rightarrow k=4m+6$

K: $mx-y+4m+6=0$

3 (b) (ii)

> To plot a straight line you need 2 points. Two good points to choose are where the line cuts the axes.
> 1. Crosses the X-axis: Put $y = 0$ (, 0)
> 2. Crosses the Y-axis: Put $x = 0$ (0,)

X intercept: $mx-(0)+4m+6=0 \Rightarrow mx=-4m-6 \Rightarrow x=\dfrac{-4m-6}{m}$

Y intercept: $m(0)-y+4m+6=0 \Rightarrow y=4m+6$

Intercepts: $\left(\dfrac{-4m-6}{m}, 0\right)$, $(0, 4m+6)$

3 (b) (iii)

$$A=\tfrac{1}{2}\left|\left(\frac{-4m-6}{m}\right)(4m+6)-(0)(0)\right|=54$$

$A=\tfrac{1}{2}|x_1y_2-x_2y_1|$ 4

$$\Rightarrow\left|-\left(\frac{4m+6}{m}\right)(2m+3)\right|=54\Rightarrow(4m+6)(2m+3)=54m$$

$$\Rightarrow(2m+3)(2m+3)=27m\Rightarrow4m^2+12m+9=27m$$

$$\Rightarrow4m^2-15m+9=0\Rightarrow(4m-3)(m-3)=0\Rightarrow m=\tfrac{3}{4},\ 3$$

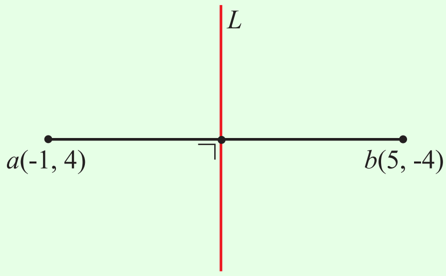
2002

3 (a) $a(-1,\ 4)$ and $b(5,\ -4)$ are two points. Find the equation of the perpendicular bisector of $[ab]$.

SOLUTION

3 (a)

The perpendicular bisector L passes through the midpoint of $[ab]$ and has a perpendicular slope to ab.



Midpoint of $[ab]$: $\left(\frac{-1+5}{2},\ \frac{4-4}{2}\right)=(2,\ 0)$

$m=\tan\theta=\dfrac{y_2-y_1}{x_2-x_1}$ 2

2. **PERPENDICULAR LINES**
Two lines are perpendicular if the product of their slopes is -1.
$K\perp L\Leftrightarrow m_1\times m_2=-1$. If $m_1=-\frac{a}{b}\Rightarrow m_2=\frac{b}{a}$

Slope of ab: $m_1=\dfrac{4+4}{-1-5}=-\dfrac{8}{6}=-\dfrac{4}{3}$

Therefore, slope of L: $m_2=\dfrac{3}{4}$

Equation of a line: $y-y_1=m(x-x_1)$ 3

Equation of L: $3x-4y+k=0\Rightarrow3(2)-4(0)+k=0\Rightarrow k=-6$

L: $3x-4y-6=0$

2001

3 (a) The line B contains the points $(6, -2)$ and $(-4, 10)$. The line A with equation $ax + 6y + 21 = 0$ is perpendicular to B. Find the value of the real number a.

SOLUTION

3 (a)

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{.......} \ \textbf{2}$$

2. **PERPENDICULAR LINES**
Two lines are perpendicular if the product of their slopes is -1.
$K \perp L \Leftrightarrow m_1 \times m_2 = -1$. If $m_1 = -\frac{a}{b} \Rightarrow m_2 = \frac{b}{a}$

Slope of B: $m_1 = \dfrac{-2-10}{6+4} = -\dfrac{12}{10} = -\dfrac{6}{5}$

Slope of A: $m_2 = -\dfrac{a}{6}$

$$\therefore m_1 \times m_2 = \left(-\frac{6}{5}\right)\left(-\frac{a}{6}\right) = -1 \Rightarrow \frac{a}{5} = -1 \Rightarrow a = -5$$