LINE (Q 3, PAPER 2)

2007

3 (a) Find the area of the triangle with vertices $(1, 1)$, $(8, -5)$ and $(5, -2)$.

(b) f is the transformation $(x, y) \to (x', y')$, where $x' = 4x + 2y$ and $y' = -3x - y$.
K is the line $x + y = 0$.
(i) Show that K is its own image under f.

(ii) $p(1, -1)$ and $q(3, -3)$ are two points.

Find the ratio $|pq| : |f(p)f(q)|$, giving your answer in its simplest form.

(c) Consider the equation $k(3x - 5y + 6) + l(5x - 7y + 4) = 0$, where $k, l \in \mathbf{R}$.
(i) Show that for all k and l, the given equation represents a line passing through the point of intersection of $3x - 5y + 6 = 0$ and $5x - 7y + 4 = 0$.

(ii) Find the relationship between k and l for which the given equation represents a line of slope 2.

(iii) If $k = 1$, what line through the point of intersection cannot be represented by the given equation? Justify your answer.

SOLUTION

3 (a)

$(1,1) \to (0,0)$
$(8,-5) \to (7,-6)$
$(5,-2) \to (4,-3)$

$A = \frac{1}{2}|(7)(-3) - (-6)(4)| = \frac{1}{2}|-21 + 24| = \frac{3}{2}$

$A = \frac{1}{2}|x_1y_2 - x_2y_1|$ 4

STEPS
1. Translate one point to (0, 0).
2. Do the same translation to the other two points.
3. Apply the formula.

3 (b)

$x' = 4x + 2y$ ⇐ $x' = 4x + 2y$ ⇒ $3x' = 12x + 6y$

$2y' = -6x - 2y$ ⇐ $y' = -3x - y$ ⇒ $4y' = -12x - 4y$

$\frac{-x' - 2y'}{2} = x$ $\qquad$ $\frac{3x' + 4y'}{2} = y$

3 (b) (i)

$K: x + y = 0$

$K': \frac{-x' - 2y'}{2} + \frac{3x' + 4y'}{2} = 0 \Rightarrow -x' - 2y' + 3x' + 4y' = 0 \Rightarrow 2x' + 2y' = 0$

$\Rightarrow x' + y' = 0$

Therefore, K is its own image under f.

3 (b) (ii)

$p(1,-1) \Rightarrow f(p) = (2,-2)$

$q(3,-3) \Rightarrow f(q) = (6,-6)$

$|pq| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

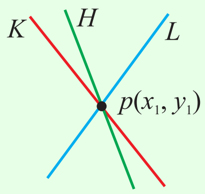
$|f(p)f(q)| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\Rightarrow |pq| : |f(p)f(q)| = 2\sqrt{2} : 4\sqrt{2} = 1:2$

3 (c)

$k(3x-5y+6)+l(5x-7y+4)=0 \Rightarrow (3k+5l)x+(-5k-7l)y+(6k+4l)=0$ ....Eqn. **1**

CONCURRENT LINE FORMULA: Given any two concurrent lines, K and L, any other line, H, concurrent with these can be expressed as: $H = \mu K + \lambda L$ where $\mu, \lambda \in \mathbf{R}$.



3 (c) (i)

Find the point of intersection of $3x-5y+6=0$ and $5x-7y+4=0$.

$3x-5y+6=0\,(\times 5) \rightarrow 15x-25y+30=0$

$5x-7y+4=0\,(\times -3) \rightarrow -15x+21y-12=0$

$-4y+18=0 \Rightarrow y = \frac{9}{2}$

$\therefore 3x-5(\frac{9}{2})+6=0 \Rightarrow 3x=\frac{33}{2} \Rightarrow x=\frac{11}{2}$

Point of intersection: $(\frac{11}{2}, \frac{9}{2})$

Substitute this point into Eqn. **1**:

$\therefore (3k+5l)(\frac{11}{2})+(-5k-7l)(\frac{9}{2})+(6k+4l) = \frac{33}{2}k+\frac{55}{2}l-\frac{45}{2}k-\frac{63}{2}l+6k+4l=0$

Therefore, $k(3x-5y+6)+l(5x-7y+4)=0$, where $k, l \in \mathbf{R}$, represents the equation of a line passing through $(\frac{11}{2}, \frac{9}{2})$ for all values of k and l.

3 (c) (ii)

Slope of Eqn. **1**, $m = -\frac{3k+5l}{-5k-7l} = \frac{3k+5l}{5k+7l} = 2 \Rightarrow 3k+5l = 10k+14l$

$\Rightarrow 7k+9l=0$

3 (c) (iii)

The line $5x-7y+4=0$ is generated from Eqn. **1** for values of k = 0 and any value of l. If k = 1, then this line which contains the point of intersection cannot be produced from Eqn. **1**.