LINE (Q 3, PAPER 2)

2004

3 (a) $a(-1, 4)$ and $b(9, -1)$ are two points and p is a point in $[ab]$. Given that $|ap| : |pb| = 2 : 3$, find the co-ordinates of p.

3 (b) (i) Calculate the perpendicular distance from the point $(-1, -5)$ to the line $3x - 4y - 2 = 0$.

(ii) The point $(-1, -5)$ is equidistant from the lines $3x - 4y - 2 = 0$ and $3x - 4y + k = 0$, where $k \neq -2$. Find the value of k.

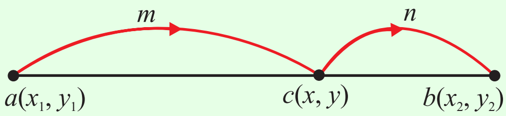
3 (c) f is the transformation $(x, y) \to (x', y')$, where $x' = 2x - y$ and $y' = x + y$. L is the line $y = mx + c$. K is the line through the origin that is perpendicular to L.

(i) Find the equation of $f(L)$ and the equation of $f(K)$.

(ii) Find the values of m for which $f(K) \perp f(L)$. Give your answer in surd form.

SOLUTION

3 (a)



$$x = \frac{mx_2 + nx_1}{m+n},\ y = \frac{my_2 + ny_1}{m+n}$$ 5

$$x = \frac{2(9) + 3(-1)}{5} = 3,\ y = \frac{2(-1) + 3(4)}{5} = 2$$

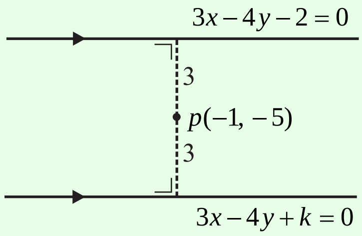
Ans: $p(3, 2)$

3 (b) (i)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$ 8

$$d = \frac{|3(-1) - 4(-5) - 2|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 20 - 2|}{\sqrt{9+16}} = \frac{|15|}{\sqrt{25}} = 3$$

3 (b) (ii)



$$3 = \frac{|3(-1) - 4(-5) + k|}{\sqrt{9+16}} \Rightarrow 3 = \frac{|k + 17|}{5} \Rightarrow 15 = |k + 17|$$

$$\Rightarrow \pm 15 = k + 17 \Rightarrow k = -32, -2$$

Ans: $k = -32$

3 (c)

L: $mx - y + c = 0$, Slope $= +\frac{m}{1}$

K: (0, 0), Slope $= -\frac{1}{m}$

Equation of K: $x + my + k = 0 \Rightarrow (0) + m(0) + k = 0 \Rightarrow k = 0$

K: $x + my = 0$

3 (c) (i)

$$x' = 2x - y \qquad \Leftarrow x' = 2x - y \Rightarrow \qquad x' = 2x - y$$

$$y' = x + y \qquad \Leftarrow y' = x + y \Rightarrow \qquad -2y' = -2x - 2y$$

$$\frac{x' + y'}{3} = x \qquad\qquad -\frac{x' - 2y'}{3} = y$$

$$f(L):\ m\left(\frac{x' + y'}{3}\right) - \left(-\frac{x' - 2y'}{3}\right) + c = 0 \Rightarrow mx' + my' + x' - 2y' + 3c = 0$$

$$\Rightarrow (m+1)x' + (m-2)y' + 3c = 0$$

$$f(K):\ \left(\frac{x' + y'}{3}\right) + m\left(-\frac{x' - 2y'}{3}\right) = 0 \Rightarrow x' + y' - mx' + 2my' = 0$$

$$\Rightarrow (1-m)x' + (2m+1)y' = 0$$

3 (c) (ii)

> 2. **Perpendicular lines**
> Two lines are perpendicular if the product of their slopes is -1.
> $K \perp L \Leftrightarrow m_1 \times m_2 = -1$. If $m_1 = -\frac{a}{b} \Rightarrow m_2 = \frac{b}{a}$

$$\left(-\frac{1-m}{2m+1}\right)\left(-\frac{m+1}{m-2}\right) = -1 \Rightarrow (1-m)(m+1) = -1(2m+1)(m-2)$$

$$\Rightarrow m^2 - 3m - 1 = 0 \Rightarrow m = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2} = \frac{3 \pm \sqrt{9+4}}{2}$$

$$\Rightarrow m = \frac{3 \pm \sqrt{13}}{2}$$