

LINE (Q 3, PAPER 2)

2003

3 (a) f is the transformation $(x, y) \rightarrow (x', y')$ where $x' = x + y$ and $y' = x - y$. L is the line $4x - 2y - 1 = 0$. Find the equation of $f(L)$, the image of L under f .

3 (b) K is the line $3x - 4y + 9 = 0$. The point $a(-3, 0)$ is on K .

The line M is parallel to K . The point $p(2, -1)$ is midway between K and M .

(i) Find the equation of M .

(ii) Calculate the distance between K and M .

(iii) Calculate the measure of the acute angle between ap and K . Give your answer correct to the nearest degree.

(iv) $b(x, y)$ is a point on K such that $|ab| = 15$ and $x > 0$. Find the value of x and the value of y .



SOLUTION

3 (a)

$$\begin{array}{lll} x' = x + y & \Leftarrow x' = x + y \Rightarrow & x = x + y \\ \underline{y' = x - y} & \Leftarrow y' = x - y \Rightarrow & \underline{-y' = -x + y} \\ \frac{x' + y'}{2} = x & & \frac{x' - y'}{2} = y \end{array}$$

$$f(L): 4x - 2y - 1 = 0 \Rightarrow 4\left(\frac{x' + y'}{2}\right) - 2\left(\frac{x' - y'}{2}\right) - 1 = 0$$

$$\Rightarrow 2(x' + y') - (x' - y') - 1 = 0 \Rightarrow 2x' + 2y' - x' + y' - 1 = 0$$

$$\Rightarrow x' + 3y' - 1 = 0$$

3 (b) (i)

Find a point on M by translating the point a through p .

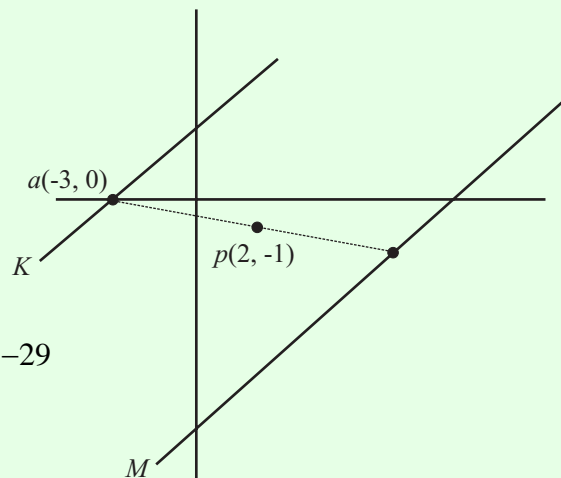
$$a(-3, 0) \rightarrow p(2, -1) \rightarrow (7, -2)$$

M has the same slope as K .

$$\text{Equation of } M: m = \frac{3}{4}, (7, -2)$$

$$\therefore 3x - 4y + k = 0 \Rightarrow 3(7) - 4(-2) + k = 0 \Rightarrow k = -29$$

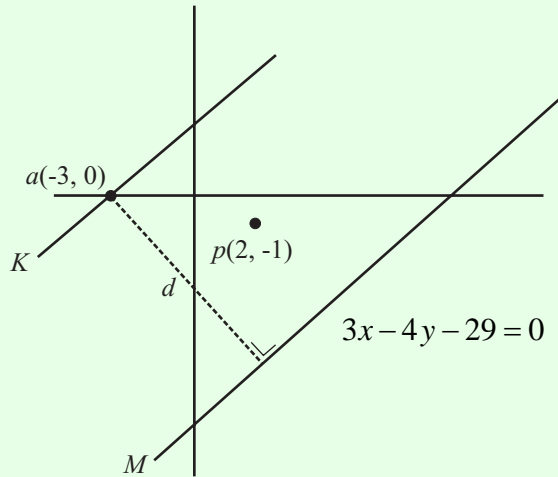
$$M: 3x - 4y - 29 = 0$$



3 (b) (ii)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \dots\dots \textcircled{8}$$

$$d = \frac{|3(-3) - 4(0) - 29|}{\sqrt{9+16}} = \frac{|-9+0-29|}{5} = \frac{38}{5}$$



3 (b) (iii)

$$\tan \theta = \pm \left(\frac{m_1 - m_2}{1 + m_1 m_2} \right) \dots\dots \textcircled{7}$$

Slope of ap : $m_1 = \frac{0+1}{-3-2} = -\frac{1}{5}$

Slope of K : $m_2 = \frac{3}{4}$

$$\tan \theta = \left| \frac{-\frac{1}{5} - \frac{3}{4}}{1 + (-\frac{1}{5})(\frac{3}{4})} \right| \Rightarrow \tan \theta = \left| \frac{-4-15}{20-3} \right| = \left| \frac{-19}{17} \right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{19}{17}\right) = 48^\circ$$

3 (b) (iv)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \dots\dots \textcircled{1}$$

$$15 = \sqrt{(x+3)^2 + (y-0)^2} \Rightarrow 225 = (x+3)^2 + y^2$$

$$b \in K \Rightarrow y = \frac{3x+9}{4}$$

$$\therefore 225 = (x+3)^2 + \left(\frac{3x+9}{4}\right)^2$$

$$\Rightarrow 225 = x^2 + 6x + 9 + \left(\frac{9x^2 + 54x + 81}{16}\right)$$

$$\Rightarrow 3600 = 16x^2 + 96x + 144 + 9x^2 + 54x + 81$$

$$\Rightarrow 0 = 25x^2 + 150x - 3375 \Rightarrow x^2 + 6x - 135 = 0$$

$$\Rightarrow (x-9)(x+15) = 0 \Rightarrow x = -15, 9$$

$$\text{As } x > 0 \Rightarrow x = 9, y = \frac{3(9)+9}{4} = 9$$

Ans: (9, 9)

