LINE (Q 3, PAPER 2)

2001

3 (a) The line *B* contains the points $(6, -2)$ and $(-4, 10)$. The line *A* with equation $ax + 6y + 21 = 0$ is perpendicular to *B*. Find the value of the real number *a*.

3 (b) *f* is the transformation $(x, y) \to (x', y')$

$$x' = -5x - 6y$$
$$y' = 4x + 3y.$$

L is the line $x - 9y = 2$.

(i) Find the equation of *f*(*L*), the image of *L* under *f*.

M is a line containing the point (1, *k*) where $k \in \mathbf{Z}$.

(ii) Given that *f*(*M*) is $5x' - 2y' + 3k = 0$, find the value of *k*.

3 (c) *N* is the line $tx + (t-2)y + 4 = 0$ where $t \in \mathbf{R}$.

(i) Write down the slope of *N* in terms of *t*.

(ii) Given that the angle between *N* and the line $x - 3y + 1 = 0$ is 45°, find the two possible values of *t*.

SOLUTION

3 (a)

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$ **2**

2. PERPENDICULAR LINES
Two lines are perpendicular if the product of their slopes is -1.
$K \perp L \Leftrightarrow m_1 \times m_2 = -1$. If $m_1 = -\frac{a}{b} \Rightarrow m_2 = \frac{b}{a}$

Slope of *B*: $m_1 = \dfrac{-2-10}{6+4} = -\dfrac{12}{10} = -\dfrac{6}{5}$

Slope of *A*: $m_2 = -\dfrac{a}{6}$

$$\therefore m_1 \times m_2 = \left(-\frac{6}{5}\right)\left(-\frac{a}{6}\right) = -1 \Rightarrow \frac{a}{5} = -1 \Rightarrow a = -5$$

3 (b) (i)

$$x' = -5x - 6y \quad \Leftarrow x' = -5x - 6y \Rightarrow \quad 4x' = -20x - 24y$$
$$2y' = 8x + 6y \quad \Leftarrow y' = 4x + 3y \Rightarrow \quad 5y' = 20x + 15y$$
$$\frac{x' + 2y'}{3} = x \qquad\qquad -\frac{4x' + 5y'}{9} = y$$

$$L: x - 9y - 2 = 0 \Rightarrow f(L): \left(\frac{x' + 2y'}{3}\right) - 9\left(-\frac{4x' + 5y'}{9}\right) - 2 = 0$$

$$\Rightarrow f(L): x' + 2y' + 12x' + 15y' - 6 = 0 \Rightarrow f(L): 13x' + 17y' - 6 = 0$$

3 (b) (ii)

$f(M)$: $5x' - 2y' + 3k = 0 \Rightarrow M$: $5(-5x - 6y) - 2(4x + 3y) + 3k = 0$

$\Rightarrow M$: $-25x - 30y - 8x - 6y + 3k = 0 \Rightarrow M$: $33x + 36y - 3k = 0$

$\Rightarrow M$: $11x + 12y - k = 0$

$(1, k) \in M \Rightarrow 11(1) + 12(k) - k = 0 \Rightarrow 11 + 12k - k = 0 \Rightarrow 11 + 11k = 0$

$\Rightarrow 11 = -11k \Rightarrow k = -1$

3 (c)

N: $tx + (t-2)y + 4 = 0$

3 (c) (i)

If you are given the equation of a line you can write down its slope:

$ax + by + c = 0 \Rightarrow m = -\frac{a}{b}$

Slope of N: $m_1 = -\dfrac{t}{t-2} = \dfrac{t}{2-t}$

3 (c) (ii)

$\tan\theta = \pm\left(\dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$ 7

L: $x - 3y + 1 = 0 \Rightarrow m_2 = \frac{1}{3}$

$$\tan 45^\circ = \left|\frac{\frac{t}{2-t} - \frac{1}{3}}{1 + (\frac{t}{2-t})(\frac{1}{3})}\right| \Rightarrow 1 = \left|\frac{\frac{t}{2-t} - \frac{1}{3}}{1 + (\frac{t}{2-t})(\frac{1}{3})}\right| \times \frac{3(2-t)}{3(2-t)}$$

$$\Rightarrow 1 = \left|\frac{3t - (2-t)}{3(2-t) + t}\right| \Rightarrow 1 = \left|\frac{4t - 2}{6 - 2t}\right| \Rightarrow 1 = \left|\frac{2t - 1}{3 - t}\right|$$

$$\Rightarrow \pm 1 = \frac{2t - 1}{3 - t} \Rightarrow \pm 1(3 - t) = (2t - 1)$$

$3 - t = 2t - 1 \Rightarrow 4 = 3t \Rightarrow t = \frac{4}{3}$ or $-3 + t = 2t - 1 \Rightarrow t = -2$

Ans: $t = -2, \frac{4}{3}$