LINE (Q 3, PAPER 2)

LESSON NO. 6: LINEAR TRANSFORMATIONS

2003

3 (a) f is the transformation $(x,\ y) \to (x',\ y')$ where $x' = x + y$ and $y' = x - y$. L is the line $4x - 2y - 1 = 0$. Find the equation of $f(L)$, the image of L under f.

2002

3 (b) f is the transformation $(x,\ y) \to (x',\ y')$ where $x' = 3x + y$ and $y' = x - 2y$. S is the square whose vertices are (0, 0), (1, 0), (1, 1) and (0, 1).

(i) Find the image of f of each of the four vertices of S.

(ii) Express x and y in terms of x' and y'.

(iii) By considering the lines $ax + by + c = 0$ and $ax + by + d = 0$, or otherwise, prove that f maps every pair of parallel lines. (You may assume that f maps every line to a line.)

(iv) Show both S and $f(S)$ on a diagram.

(v) Find the area of $f(S)$.

2001

3 (b) f is the transformation $(x,\ y) \to (x',\ y')$

$$x' = -5x - 6y$$
$$y' = 4x + 3y.$$

L is the line $x - 9y = 2$.

(i) Find the equation of $f(L)$, the image of L under f.

M is a line containing the point $(1, k)$ where $k \in \mathbf{Z}$.

(ii) Given that $f(M)$ is $5x' - 2y' + 3k = 0$, find the value of k.

2005

3 (c) f is the transformation $(x,\ y) \to (x',\ y')$, where $x' = 3x - y$ and $y' = x + 2y$.

(i) Prove that f maps every pair of parallel lines to a pair of parallel lines. You may assume that f maps every line to a line.

(ii) $oabc$ is a parallelogram, where $[ob]$ is a diagonal and o is the origin. Given that $f(c) = (-1,\ 9)$, find the slope of ab.

2004

3 (c) f is the transformation $(x, y) \to (x', y')$, where $x' = 2x - y$ and $y' = x + y$. L is the line $y = mx + c$. K is the line through the origin that is perpendicular to L.

(i) Find the equation of $f(L)$ and the equation of $f(K)$.

(ii) Find the values of m for which $f(K) \perp f(L)$. Give your answer in surd form.

2006

3 (c) (ii) L is the line $y = 4x$ and K is the line $x = 4y$. f is the transformation $(x, y) \to (x', y')$, where $x' = 2x - y$ and $y' = x + 3y$. Find the measure of the acute angle between $f(L)$ and $f(K)$, correct to the nearest degree.

Answers

2003 3 (a) $f(L) = x' + 3y' - 1 = 0$

2002 3 (b) (i) $(0, 0), (3, 1), (4, -1), (1, -2)$ (ii) $x = \frac{1}{7}(2x' + y'),\ y = \frac{1}{7}(x' - 3y')$
(v) 7

2001 3 (b) (i) $f(L) = 13x' + 17y' - 6 = 0$ (ii) $k = -1$

2005 3 (c) (i) 4

2004 3 (c) (i) $f(L) = (m+1)x' + (m-2)y' + 3c = 0;\ f(K) = (1-m)x' + (2m+1)y' = 0$
(ii) $m = \frac{1}{2}(3 \pm \sqrt{13})$

2006 3 (c) (ii) 54°