

INTEGRATION (Q 8, PAPER 1)

LESSON NO. 8: APPLICATIONS OF INTEGRATION II: VOLUME

2005

8 (c) (ii) Use integration methods to derive a formula for the volume of a cone.

SOLUTION

Ex. Use integration methods to derive a formula for the volume of a cone.

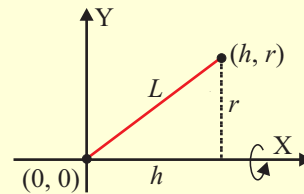
SOLUTION

Slope of $L = \frac{r}{h} \Rightarrow$ Equation of $L: rx - hy = 0 \Rightarrow y = \frac{r}{h}x$

Volume of cone $V = \pi \int_0^r y^2 dx$

$$= \pi \frac{r^2}{h^2} \int_0^h x^2 dx = \pi \frac{r^2}{3h^2} [x^3]_0^h = \pi \frac{r^2}{3h^2} [h^3 - 0]$$

$$= \frac{1}{3} \pi r^2 h$$



2003

8 (c) (ii) Use integration methods to show that the volume of a sphere with radius r is $\frac{4}{3} \pi r^3$.

SOLUTION

Use integration methods to derive a formula for the volume of a sphere.

SOLUTION

Rotate the circle C with centre $(0, 0)$ and radius r about the X -axis.

Equation of $C: x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2$

Volume of sphere $V = \pi \int_{-r}^r y^2 dx = \pi \int_{-r}^r (r^2 - x^2) dx$

$$= \pi [r^2 x - \frac{1}{3} x^3]_{-r}^r = \pi [(r^3 - \frac{1}{3} r^3) - (-r^3 + \frac{1}{3} r^3)] = \frac{4}{3} \pi r^3$$

