INTEGRATION (Q 8, PAPER 1)

LESSON NO. 6: SPECIALS

2001

8 (b) Evaluate (i) $\int_0^3 \frac{12}{x^2+9}\,dx$

SOLUTION

$$\int \frac{dx}{(a)^2+(x)^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c \quad \text{.......} \; \mathbf{6}$$

$$I = \int_0^3 \frac{12}{x^2+9}\,dx = 12\int_0^3 \frac{1}{(x)^2+(3)^2}\,dx = \tfrac{12}{3}[\tan^{-1}\tfrac{x}{3}]_0^3$$

$$= 4[\tan^{-1}1 - \tan^{-1}0] = 4[\tfrac{\pi}{4}-0] = \pi$$

2002

8 (b) Evaluate (ii) $\int_2^7 \frac{1}{x^2-4x+29}\,dx.$

SOLUTION

$$I = \int_2^7 \frac{1}{x^2-4x+29}\,dx$$

$$\int \frac{dx}{(a)^2+(x\pm b)^2} = \frac{1}{a}\tan^{-1}\left(\frac{x\pm b}{a}\right)+c \quad \text{.......} \; \mathbf{6}$$

Firstly, sort out the quadratic: $x^2-4x+29 = x^2-4x+4+25 = (x-2)^2+(5)^2$

$$\therefore I = \int_2^7 \frac{1}{(x-2)^2+(5)^2}\,dx = \left[\frac{1}{5}\tan^{-1}\left(\frac{x-2}{5}\right)\right]_2^7$$

$$= \tfrac{1}{5}[\tan^{-1}1 - \tan^{-1}0] = \tfrac{1}{5}[\tfrac{\pi}{4}-0] = \tfrac{\pi}{20}$$

2004

8 (b) Evaluate (i) $\int_3^6 \frac{dx}{\sqrt{36-x^2}}$

SOLUTION

$$\int \frac{dx}{\sqrt{(a)^2-(x)^2}} = \sin^{-1}\left(\frac{x}{a}\right)+c \quad \text{.......} \; \mathbf{7}$$

$$\int_3^6 \frac{dx}{\sqrt{36-x^2}} = \int_3^6 \frac{dx}{\sqrt{(6)^2-(x)^2}} = [\sin^{-1}\tfrac{x}{6}]_3^6$$

$$= [\sin^{-1}1 - \sin^{-1}\tfrac{1}{2}] = \tfrac{\pi}{2}-\tfrac{\pi}{6} = \tfrac{\pi}{3}$$

2005

8 (c) (i) Evaluate $\int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx.$

Solution

$$I = \int_1^2 \frac{1}{\sqrt{3+2x-x^2}}\,dx$$

$$\int \frac{dx}{\sqrt{(a)^2-(x\pm b)^2}} = \sin^{-1}\left(\frac{x\pm b}{a}\right)+c$$ 7

Start by working on the quadratic:

$$3+2x-x^2 = -(x^2-2x-3) = -(x^2-2x+1-4)$$

$$= -((x-1)^2-2^2) = (2)^2-(x-1)^2$$

$$= \left[\sin^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(\frac{0}{2}\right)\right] = \sin^{-1}\tfrac{1}{2} = \tfrac{\pi}{6}$$