INTEGRATION (Q 8, PAPER 1)

LESSON NO. 5: TRIGONOMETRIC INTEGRATION II

2005

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{8}} \sin^2 2\theta \, d\theta$

SOLUTION

$$I = \int_0^{\frac{\pi}{8}} \sin^2 2\theta \, d\theta$$

Trig squares are dealt with by using these formulae:

$\sin^2 A = \frac{1}{2}(1-\cos 2A)$

$\cos^2 A = \frac{1}{2}(1+\cos 2A)$

$$\therefore I = \frac{1}{2}\int_0^{\frac{\pi}{8}} (1-\cos 4\theta)\, d\theta = \frac{1}{2}[\theta - \frac{1}{4}\sin 4\theta]_0^{\frac{\pi}{8}}$$

$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c$ 5

$$= \frac{1}{2}[(\frac{\pi}{8} - \frac{1}{4}\sin\frac{\pi}{2}) - (0 - \frac{1}{4}\sin 0)] = \frac{1}{2}[(\frac{\pi}{8} - \frac{1}{4}) - (0-0)] = \frac{\pi}{16} - \frac{1}{8}$$

2003

8 (b) (ii) By letting $u = \sin x$, evaluate $\int_0^{\frac{\pi}{2}} \cos x \sin^6 x \, dx$.

SOLUTION

$$I = \int_0^{\frac{\pi}{2}} \cos x \sin^6 x \, dx$$

EVEN AND ODD POWER: These are integrals that are products of a trig function with an even power and an odd power. They are done by substitution.

STEPS
1. Bracket the EVEN power trig function.
2. Break off one of the odd power trig functions and bracket it with *dx*.
3. Let *u* equal the trig function with the even power.
4. Integrate by substitution as already explained.

1/2. $I = \int_0^{\frac{\pi}{2}} (\sin x)^6 (\cos x \, dx)$

3. Let $u = \sin x \Rightarrow du = \cos x \, dx$

Changing limits:
$x = \frac{\pi}{2} \Rightarrow u = \sin\frac{\pi}{2} = 1$
$x = 0 \Rightarrow u = \sin 0 = 0$

4. $\therefore I = \int_0^1 u^6 du = \frac{1}{7}[u^7]_0^1 = \frac{1}{7}[1-0] = \frac{1}{7}$

2004

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{3}} \sin x \cos^3 x \, dx$

SOLUTION

$$I = \int_0^{\frac{\pi}{3}} \sin x \cos^3 x \, dx$$

TWO ODD POWERS

STEPS
1. Bracket the **HIGHER** powered trig function.
2. Break off one of the other trig functions and bracket with dx.
3. Let u equal the trig function with the higher power.
4. Integrate by substitution as already explained.

1/2. $I = \int_0^{\frac{\pi}{3}} (\cos x)^3 (\sin x \, dx)$

3. Let $u = \cos x \Rightarrow du = -\sin x \, dx \Rightarrow -du = \sin x \, dx$

Changing limits:

$x = \frac{\pi}{3} \Rightarrow u = \cos \frac{\pi}{3} = \frac{1}{2}$

$x = 0 \Rightarrow u = \cos 0 = 1$

4. $\therefore I = -\int_1^{\frac{1}{2}} u^3 du = -\frac{1}{4}[u^4]_1^{\frac{1}{2}} = -\frac{1}{4}[(\frac{1}{2})^4 - (1)^4] = -\frac{1}{4}[\frac{1}{16} - 1] = -\frac{1}{4}[-\frac{15}{16}] = \frac{15}{64}$