INTEGRATION (Q 8, PAPER 1)

LESSON NO. 4: TRIGONOMETRIC INTEGRATION I

2004

8 (a) Find (ii) $\int \cos 6x\, dx$

SOLUTION

$\int \cos(ax+b)dx = \frac{1}{a}\sin(ax+b)+c$ 5

$\int \cos 6x\, dx = \frac{1}{6}\sin 6x + c$

2001

8 (a) Find (ii) $\int \sin 5x\, dx$.

SOLUTION

$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$ 4

$\int \sin 5x dx = -\frac{1}{5}\cos 5x + c$

2003

8 (c) (i) Show that $\int_a^{2a} \sin 2x\, dx = \sin 3a \sin a$.

SOLUTION

$\int_a^{2a} \sin 2x\, dx = -\frac{1}{2}[\cos 2x]_a^{2a} = -\frac{1}{2}[\cos 4a - \cos 2a]$

$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b)+c$ 4

$= -\frac{1}{2}[-2\sin 3a \sin a] = \sin 3a \sin a$

SUMS → PRODUCTS
$\sin A + \sin B = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\sin A - \sin B = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$
$\cos A + \cos B = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$
$\cos A - \cos B = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

2006

8 (b) Evaluate (ii) $\int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta$.

SOLUTION

$$I = \int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta$$

PRODUCTS → SUMS
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$
$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

STEPS
1. Change products into sums using the formulae on page 9.
2. Integrate each part.

1. $I = \int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin 8\theta + \sin 2\theta) \, d\theta$

$\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$ 4

2. $I = \int_0^{\frac{\pi}{4}} \sin 5\theta \cos 3\theta \, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin 8\theta + \sin 2\theta) \, d\theta$

$$= \tfrac{1}{2}[-\tfrac{1}{8}\cos 8\theta - \tfrac{1}{2}\cos 2\theta]_0^{\frac{\pi}{4}} = -\tfrac{1}{4}[\tfrac{1}{4}\cos 8\theta + \cos 2\theta]_0^{\frac{\pi}{4}}$$

$$= -\tfrac{1}{4}[(\tfrac{1}{4}\cos 2\pi + \cos \tfrac{\pi}{2}) - (\tfrac{1}{4}\cos 0 + \cos 0)]$$

$$= -\tfrac{1}{4}[(\tfrac{1}{4}(1) + 0) - (\tfrac{1}{4}(1) + 1)] = \tfrac{1}{4}$$