INTEGRATION (Q 8, PAPER 1)

LESSON NO. 2: ALGEBRAIC INTEGRATION BY SUBSTITUTION

2006

8 (b) Evaluate (i) $\int_1^2 x(1+x^2)^3 dx$

SOLUTION

$I = \int_1^2 x(1+x^2)\, dx$

STEPS
1. Let u equal the more complicated function.
2. Differentiate u with respect to x.
3. Change the limits from x to u.
4. Make the substitution.
5. Evaluate the integral I.

1. Let $u = 1 + x^2$.

2. $du = 2x\, dx \Rightarrow \frac{1}{2} du = x\, dx$

3. $x = 1 \Rightarrow u = 1 + (1)^2 = 2$ and $x = 2 \Rightarrow u = 1 + (2)^2 = 5$

4. $I = \frac{1}{2} \int_2^5 u^3 du$

5. $\therefore I = \frac{1}{2}[\frac{1}{4}u^4]_2^5 = \frac{1}{8}[u^4]_2^5 = \frac{1}{8}[625 - 16] = \frac{609}{8}$

2005

8 (b) Evaluate (i) $\int_1^4 \frac{2x+1}{x^2+x+1} dx$

SOLUTION

STEPS
1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x.
3. Change the limits from x to u.
4. Make the substitution.
5. Evaluate the integral I.

$I = \int_1^4 \frac{2x+1}{x^2+x+1} dx$

1. Let $u = x^2 + x + 1$.

2. $\Rightarrow du = (2x+1)dx$

3. $x = 1 \Rightarrow u = 1^2 + 1 + 1 = 3$ and $x = 4 \Rightarrow u = 4^2 + 4 + 1 = 21$.

4. $\therefore I = \int_7^{21} \frac{du}{u}$

$\int \frac{1}{x} dx = \ln x + c$ **2**

5. $\Rightarrow I = [\ln u]_7^{21} = \ln 21 - \ln 7 = \ln \frac{21}{7} = \ln 3$

2003

8 (b) (i) Evaluate $\int_0^1 \frac{2x}{\sqrt{1+x^2}}\,dx$

SOLUTION

> **STEPS**
> **1.** Let u equal the *inside* of the more complicated function.
> **2.** Differentiate u with respect to x.
> **3.** Change the limits from x to u.
> **4.** Make the substitution.
> **5.** Evaluate the integral I.

$$I = \int_0^1 \frac{2x}{\sqrt{1+x^2}}\,dx$$

1. Let $u = 1 + x^2$

2. $\Rightarrow du = 2x\,dx$

3. $x = 0 \Rightarrow u = 1 + (0)^2 = 1$ and $x = 1 \Rightarrow u = 1 + (1)^2 = 2$

4. $\therefore I = \int_1^2 \frac{du}{\sqrt{u}} = \int_1^2 u^{-\frac{1}{2}}\,du$

5. $\Rightarrow I = \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^2 = 2[\sqrt{u}]_1^2 = 2[\sqrt{2} - 1]$

2002

8 (b) Evaluate (i) $\int_2^7 \frac{2x-4}{x^2-4x+29}\,dx$

SOLUTION

$$I = \int_2^7 \frac{2x-4}{x^2-4x+29}\,dx$$

> **STEPS**
> **1.** Let u equal the *inside* of the more complicated function.
> **2.** Differentiate u with respect to x.
> **3.** Change the limits from x to u.
> **4.** Make the substitution.
> **5.** Evaluate the integral I.

1. Let $u = x^2 - 4x + 29$

2. $\Rightarrow du = (2x - 4)dx$

3. $x = 2 \Rightarrow u = 2^2 - 4(2) + 29 = 25$ and $x = 7 \Rightarrow u = 7^2 - 4(7) + 29 = 50$

4. $\therefore I = \int_{25}^{50} \frac{du}{u}$

5. $\Rightarrow I = [\ln u]_{25}^{50} = \ln 50 - \ln 25 = \ln \frac{50}{25} = \ln 2$

2001

8 (b) Evaluate (ii) $\int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}}\,dx.$

SOLUTION

$$I = \int_0^4 \frac{(x+4)}{\sqrt{x^2+8x+1}}\,dx$$

STEPS
1. Let u equal the *inside* of the more complicated function.
2. Differentiate u with respect to x.
3. Change the limits from x to u.
4. Make the substitution.
5. Evaluate the integral I.

1. Let $u = x^2+8x+1$

2. $du = (2x+8)dx = 2(x+4)dx \Rightarrow \frac{1}{2}du = (x+4)dx$

3. $x = 0 \Rightarrow u = (0)^2+8(0)+1 = 1$ and $x = 4 \Rightarrow u = (4)^2+8(4)+1 = 49$

4. $\therefore I = \frac{1}{2}\int_1^{49} \frac{du}{\sqrt{u}} = \frac{1}{2}\int_1^{49} u^{-\frac{1}{2}}du$

5. $\Rightarrow I = \frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{49} = [u^{\frac{1}{2}}]_1^{49} = [\sqrt{49}-\sqrt{1}] = [7-1] = 6$