DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

LESSON NO. 8: HIGHER ORDER DIFFERENTIATION

2002

7 (c) Let $f(x)=\dfrac{e^x+e^{-x}}{2}$.

(i) Show that $f''(x)=f(x)$, where $f''(x)$ is the second derivative of $f(x)$.

(ii) Show that $\dfrac{f'(2x)}{f'(x)}=2f(x)$ when $x\neq 0$ and where $f'(x)$ is the first derivative of $f(x)$.

SOLUTION

7 (c) (i)

$f(x)=\frac{1}{2}(e^x+e^{-x})$

$\Rightarrow f'(x)=\frac{1}{2}(e^x-e^{-x})$

$\Rightarrow f''(x)=\frac{1}{2}(e^x+e^{-x})$

$\Rightarrow f(x)=f''(x)$

$y=e^x \Rightarrow \dfrac{dy}{dx}=e^x$ .... 7 Formula 7 can be extended to:

$y=e^{f(x)} \Rightarrow \dfrac{dy}{dx}=e^{f(x)}\times f'(x)$ 7

REMEMBER IT AS:

Repeat the whole function × Differentiation of the power.

7 (c) (ii)

Show $\dfrac{f'(2x)}{f'(x)}=2f(x)$

LHS

$$\frac{f'(2x)}{f'(x)}=\frac{\frac{1}{2}(e^{2x}-e^{-2x})}{\frac{1}{2}(e^x-e^{-x})}$$

$$=\frac{(e^x-e^{-x})(e^x+e^{-x})}{(e^x-e^{-x})}$$

$$=(e^x+e^{-x})$$

RHS

$2f(x)=2\times\frac{1}{2}(e^x+e^{-x})$

$=(e^x+e^{-x})$