DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

LESSON NO. 5: INVERSE TRIGONOMETRIC DIFFERENTIATION

2005

6 (a) Differentiate with respect to x

(ii) $\sin^{-1}\left(\frac{x}{5}\right)$.

SOLUTION

$$y = \sin^{-1} f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-f(x)^2}} \times f'(x) \quad \text{..... 9}$$

$$y = \sin^{-1}\left(\frac{x}{5}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(\frac{x}{5})^2}} \times \tfrac{1}{5} = \frac{1}{5\sqrt{1-\frac{x^2}{25}}} = \frac{1}{5\sqrt{\frac{25-x^2}{25}}} = \frac{1}{\sqrt{25-x^2}}$$

2004

6 (b) (i) Given $y = \tan^{-1} x$, find the value of $\frac{dy}{dx}$ at $x = \sqrt{2}$.

SOLUTION

$$\left(\frac{dy}{dx}\right)_{x=\sqrt{2}} = \frac{1}{1+(\sqrt{2})^2} = \frac{1}{1+2} = \frac{1}{3}$$

$$y = \tan^{-1} x \Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2} \quad \text{..... 10}$$

2003

7 (a) Differentiate with respect to x:

(ii) $\sin^{-1}(\frac{x}{5})$.

SOLUTION

$$y = \sin^{-1} f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-f(x)^2}} \times f'(x) \quad \text{..... 9}$$

$$y = \sin^{-1}\left(\frac{x}{5}\right) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(\frac{x}{5})^2}} \times \tfrac{1}{5} = \frac{1}{5\sqrt{1-\frac{x^2}{25}}} = \frac{1}{5\sqrt{\frac{25-x^2}{25}}} = \frac{1}{\sqrt{25-x^2}}$$

2002

7 (b) (i) Given that $y = \sin^{-1} 10x$, evaluate $\dfrac{dy}{dx}$ when $x = \dfrac{1}{20}$.

SOLUTION

$$y = \sin^{-1} f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1 - f(x)^2}} \times f'(x) \quad \text{.....} \ 9$$

$$y = \sin^{-1} 10x \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-(10x)^2}} \times 10 = \frac{10}{\sqrt{1-100x^2}}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{1}{20}} = \frac{10}{\sqrt{1-100(\frac{1}{20})^2}} = \frac{10}{\sqrt{1-\frac{1}{4}}} = \frac{10}{\sqrt{\frac{3}{4}}} = \frac{20}{\sqrt{3}}$$

2001

7 (b) (i) Differentiate $\tan^{-1} 7x$ with respect to x.

SOLUTION

$$y = \tan^{-1} f(x) \Rightarrow \frac{dy}{dx} = \frac{1}{1 + f(x)^2} \times f'(x) \quad \text{.....} \ 10$$

$$y = \tan^{-1} 7x \Rightarrow \frac{dy}{dx} = \frac{1}{1+(7x)^2} \times 7 = \frac{7}{1+49x^2}$$