DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2011

6. **(a)** Differentiate $\cos^2 x$ with respect to x.

(b) The equation of a curve is $y = e^{-x^2}$.

(i) Find $\frac{dy}{dx}$.

(ii) Find the co-ordinates of the turning point of the curve.

(iii) Determine whether this turning point is a local maximum or a local minimum.

(c) The function f is defined as $x \to \frac{2x}{x+1}$, where $x \in \mathbb{R} \setminus \{-1\}$.

(i) Find the equations of the asymptotes of the curve $y = f(x)$.

(ii) P and Q are distinct points on the curve $y = f(x)$.
The tangent at Q is parallel to the tangent at P.
The co-ordinates of P are (1, 1).
Find the co-ordinates of Q.

(iii) Verify that the point of intersection of the asymptotes is the midpoint of $[PQ]$.

ANSWERS

6 (a) $-2\cos x \sin x$

(b) (i) $\frac{dy}{dx} = -2xe^{-x^2}$ (ii) (0, 1) (iii) Local maximum

(c) (i) $x = -1$, $y = 2$ (ii) $Q(-3, 3)$

7. **(a)** Find the slope of the tangent to the curve $x^2 + y^3 = x - 2$ at the point (3, –2).

(b) A curve is defined by the parametric equations

$$x = \frac{t-1}{t+1} \text{ and } y = \frac{-4t}{(t+1)^2}, \text{ where } t \neq -1.$$

(i) Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

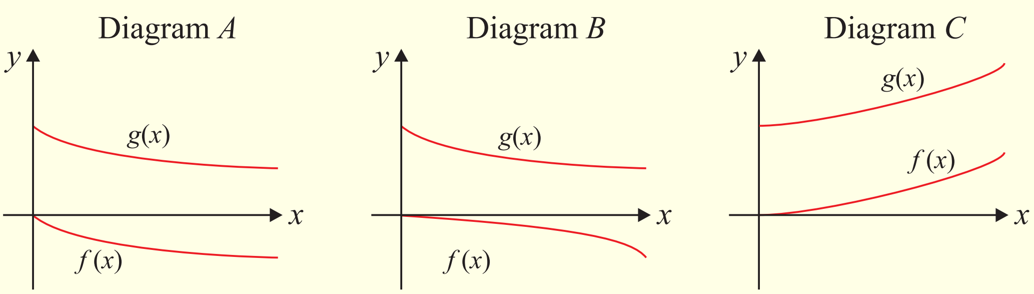
(ii) Hence find $\frac{dy}{dx}$, and express your answer in terms of x.

(c) The functions *f* and *g* are defined on the domain $x \in \mathbb{R} \setminus \{-1, 0\}$ as follows:

$$f : x \to \tan^{-1}\left(\frac{-x}{x+1}\right) \text{ and } g : x \to \tan^{-1}\left(\frac{x+1}{x}\right).$$

(i) Show that $f'(x) = \dfrac{-1}{2x^2 + 2x + 1}$.

(ii) It can be shown that $f'(x) = g'(x)$.
One of the three diagrams *A*, *B*, or *C* below represents parts of the graphs of *f* and *g*. Based only on the derivatives, state which diagram is the correct one, and state also why each of the other two diagrams is incorrect.



Answers

7 (a) $-\frac{5}{12}$

(b) (i) $\frac{dx}{dt} = \frac{2}{(t+1)^2}$, $\frac{dy}{dt} = \frac{4t-4}{(t+1)^3}$ (ii) $\frac{dy}{dx} = 2x$

(c) (ii) *A* is correct: Both functions are decreasing with the same slopes everywhere.
B is incorrect: Both slopes are not the same everywhere.
C is incorrect: Both functions are increasing.