DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2010

6 (a) The equation $x^3 + x^2 - 4 = 0$ has only one real root.

Taking $x_1 = \frac{3}{2}$ as the first approximation to the root, use the Newton-Raphson method to find x_2, the second approximation.

(b) Parametric equations of a curve are:

$$x = \frac{2t-1}{t+2}, \quad y = \frac{t}{t+2}, \text{ where } t \in \mathbf{R} \setminus \{-2\}.$$

(i) Find $\frac{dy}{dx}$.

(ii) What does your answer to part (i) tell you about the shape of the graph?

(c) A curve is defined by the equation $x^2y^3 + 4x + 2y = 12$.

(i) Find $\frac{dy}{dx}$ in terms of x and y.

(ii) Show that the tangent to the curve at the point (0, 6) is also the tangent to it at the point (3, 0).

7 (a) Differentiate x^2 with respect to x from first principles.

(b) Let $y = \frac{\cos x + \sin x}{\cos x - \sin x}$.

(i) Find $\frac{dy}{dx}$.

(ii) Show that $\frac{dy}{dx} = 1 + y^2$.

(c) The function is defined for $x > -1$.

(i) Show that the curve $f(x) = (1+x)\log_e(1+x)$ has a turning point at $\left(\frac{1-e}{e}, -\frac{1}{e}\right)$.

(ii) Determine whether the turning point is a local maximum or a local minimum.

Answers

6 (a) $\frac{4}{3}$

(b) (i) $\frac{2}{5}$ (ii) It is a straight line.

(c) (i) $\dfrac{-2xy^3-4}{3x^2y^2+2}$

7 (b) (i) $\dfrac{2}{(\cos x-\sin x)^2}$

(c) (ii) Local minimum