DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2008

6 (a) Differentiate $\sqrt{x^3}$ with respect to x.

(b) Let $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

Show that $\dfrac{dy}{dx} = \dfrac{4}{(e^x + e^{-x})^2}$.

(c) The function $f(x) = 2x^3 + 3x^2 + bx + c$ has a local maximum at $x = -2$.

(i) Find the value of b.

(ii) Find the range of values of c for which $f(x) = 0$ has three distinct real roots.

7 (a) Differentiate $2x + \sin 2x$ with respect to x.

(b) The equation of a curve is $5x^2 + 5y^2 + 6xy = 16$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y.

(ii) $(1, 1)$ and $(2, -2)$ are two points on the curve.
Show that the tangents at these points are perpendicular to each other.

(c) Let $y = \sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$.

Find $\dfrac{dy}{dx}$ and express it in the form $\dfrac{a}{a + x^b}$, where $a, b \in \mathbf{N}$.

ANSWERS

6 (a) $\frac{3}{2}\sqrt{x}$

(c) (i) $b = -12$ (ii) $-20 < c < 7$

7 (a) $2 + 2\cos 2x$

(b) (i) $-\dfrac{5x + 3y}{3x + 5y}$

(c) $\dfrac{1}{1+x^2}$