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DIFFERENTIATION & APPLICATIONS (Q 6 & 7, PAPER 1)

2007

6 (a) Differentiate $\frac{x^2-1}{x^2+1}$ with respect to x.

(b) (i) Differentiate $\frac{1}{x}$ with respect to x from first principles.

(ii) Find the equation of the tangent to $y=\frac{1}{x}$ at the point $(2, \frac{1}{2})$.

(c) Let $f(x)=\tan^{-1}\frac{x}{2}$ and $g(x)=\tan^{-1}\frac{2}{x}$, for $x>0$.

(i) Find $f'(x)$ and $g'(x)$.

(ii) Hence, show that $f(x)$ and $g(x)$ is constant.

(iii) Find the value of $f(x)+g(x)$.

7 (a) Taking 1 as the first approximation of a root of $x^3+2x-4=0$, use the Newton-Raphson method to calculate the second approximation of this root.

(b) (i) Find the equation of the tangent to the curve $3x^2+y^2=28$ at the point $(2,-4)$.

(ii) $x=e^t\cos t$ and $y=e^t\sin t$. Show that $\frac{dy}{dx}=\frac{x+y}{x-y}$.

(c) $f(x)=\log_e 3x-3x$, where $x>0$.

(i) Show that $(\frac{1}{3},-1)$ is a local maximum point of $f(x)$.

(ii) Deduce that the graph of $f(x)$ does not intersect the x-axis.

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ANSWERS

6 (a) $\dfrac{4x}{(x^2+1)^2}$

(b) (ii) $x+4y-4=0$

(c) (i) $f'(x)=\dfrac{2}{x^2+4},\ g'(x)=-\dfrac{2}{x^2+4}$

(ii) $f'(x)+g'(x)=0 \Rightarrow f(x)+g(x)=c,$ a constant. When you differentiate a constant, you get zero.

(iii) $\frac{\pi}{2}$

7 (a) $\frac{6}{5}$

(b) (i) $3x-2y-14=0$